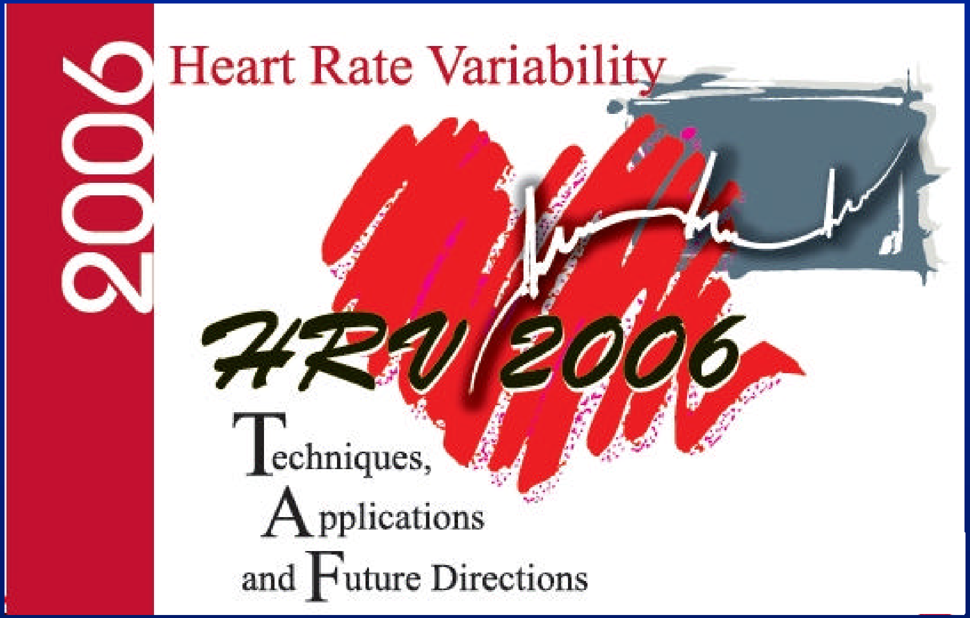
Complexity Measures: Fractal Methods



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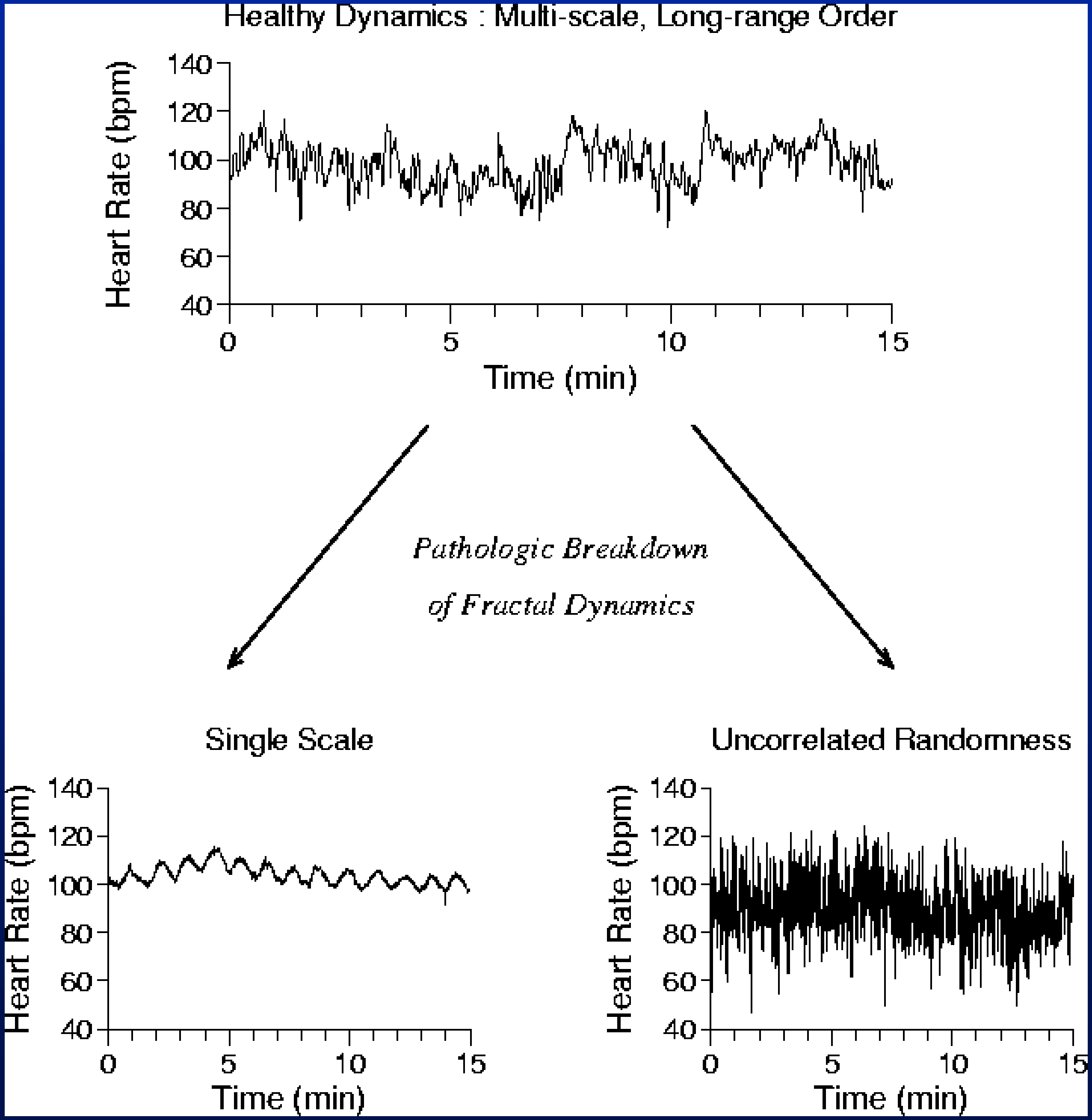
Different Roles of HRV

- To measure physiologic aspects of autonomic control
- To quantify statistical features of time series
- To characterize the dynamical properties of the underlying control systems


Healthy Dynamics : Multi-scale, Long-range Order
Heart Rate (bpm)
140
120
100
80
60
40
0
5
10
15
Time (min)
Pathologic Breakdown
of Fractal Dynamics
Single Scale
Heart Rate (bpm)
Time (min)
Uncorrelated Randomness
Heart Rate (bpm)
Time (min)

Objectives

- To introduce the concept of fractals for spatial and temporal structures
- To introduce two simple measurements of fractal objects and processes
- To discuss clinical implications of fractal dynamics in heart rate time series

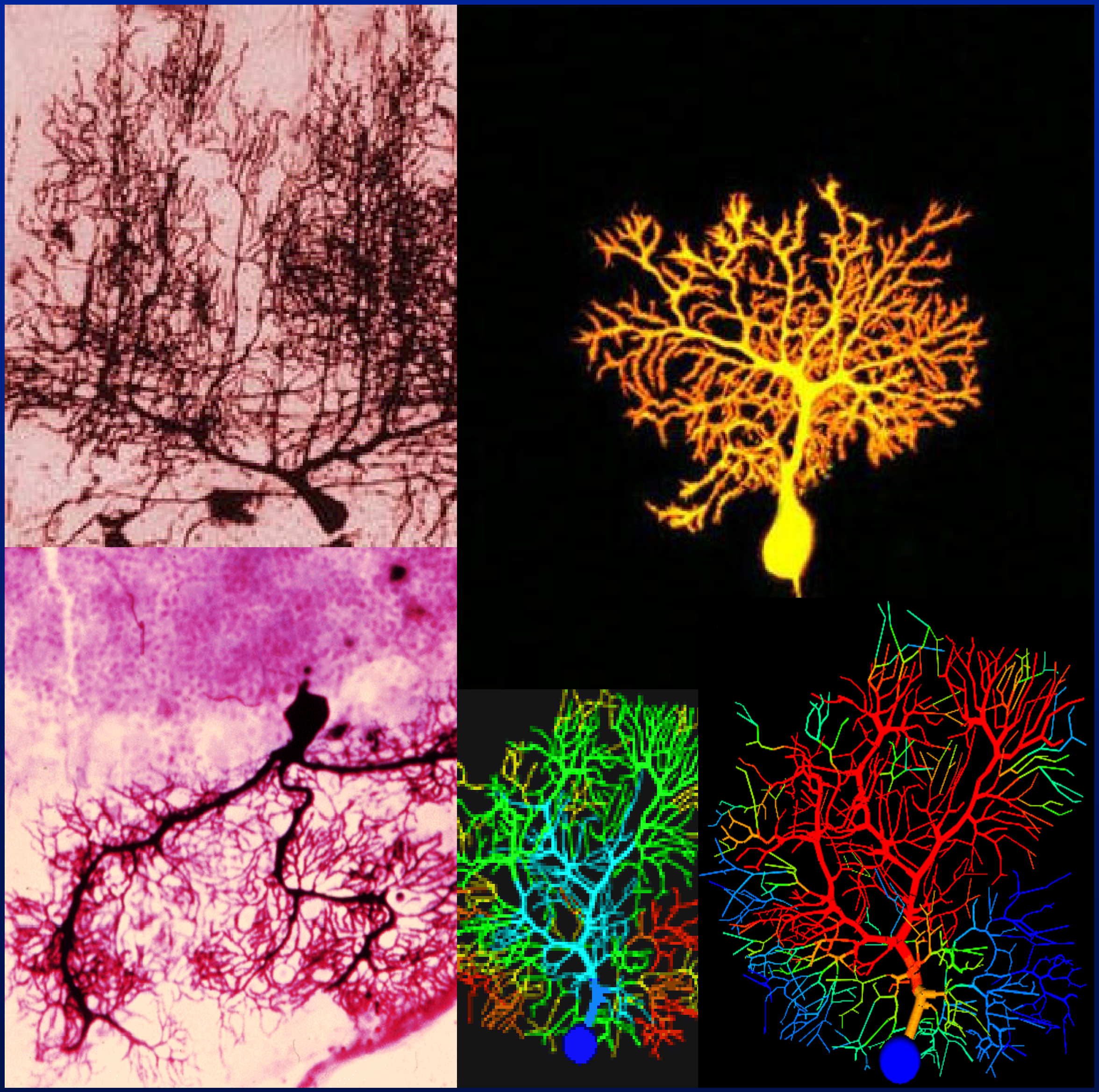
What is Fractal?

A fractal object is ***self-similar***, i.e., small subsets of the object resemble (statistically) the whole. Fractal objects do not possess a characteristic (single) spatial and temporal scale

Fractals Everywhere

- Spatial structures: tree, lung, coral, …
- Temporal dynamics: weather temperature, music, volatility of stock prices, …
- Symbolic sequences: DNA, computer codes, …

Fractal Self-Organization: Purkinje Cells in Cerebellum



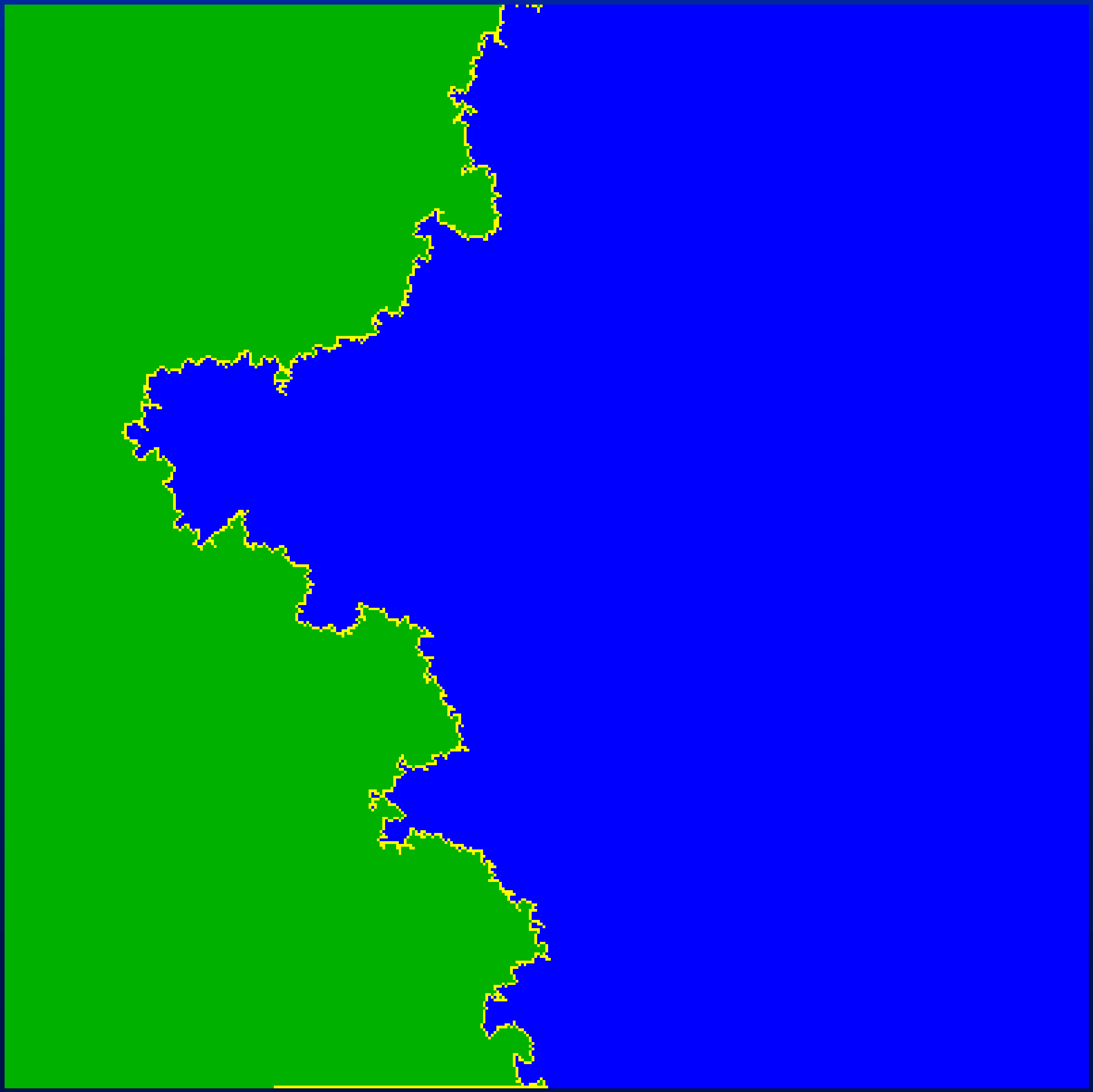
How to Measure Fractals?

Quantifying fractal with *fractal dimension* or *self-similarity parameter*

- Fractal dimension is more suitable to describe how a geometrical object fills up the space from small to large scales
- Self-similarity parameter can be used to quantify fractal processes (time series)

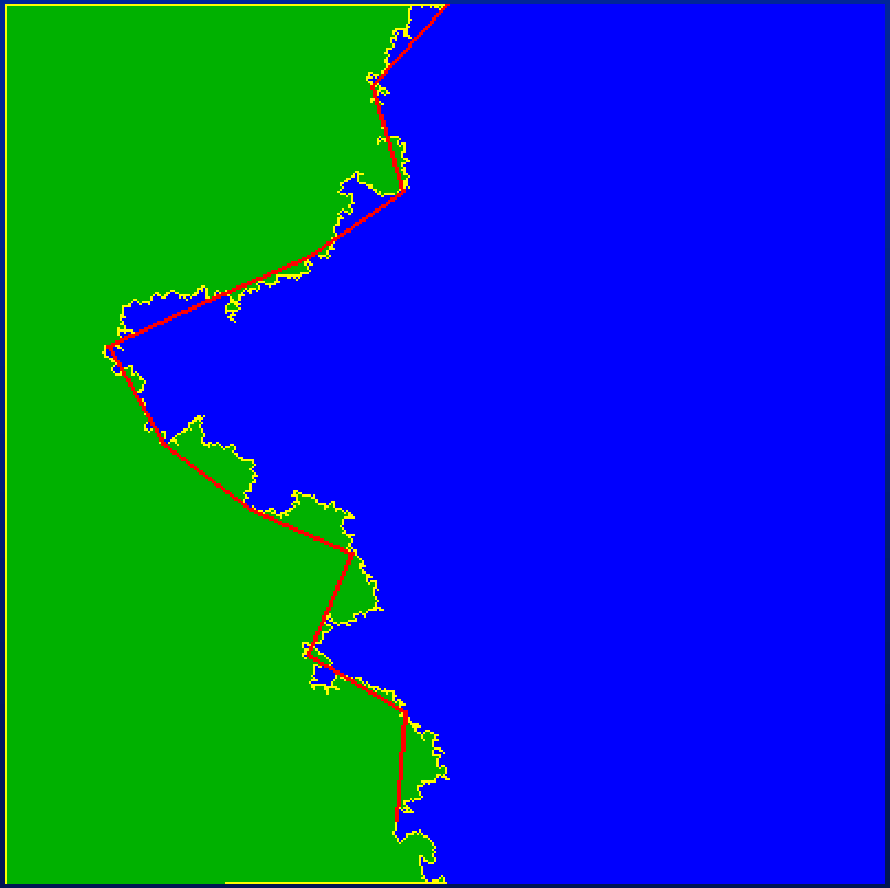
Example 1: Box counting method to measure fractal dimension

Measuring Coastline

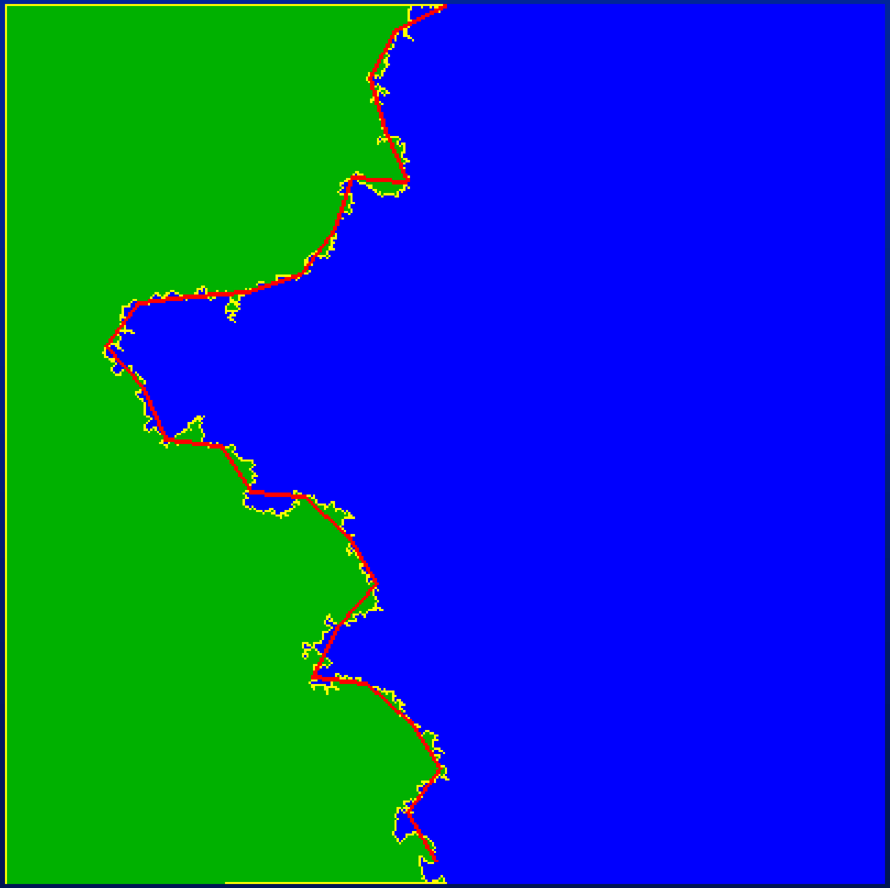


Measuring the length of a coastline

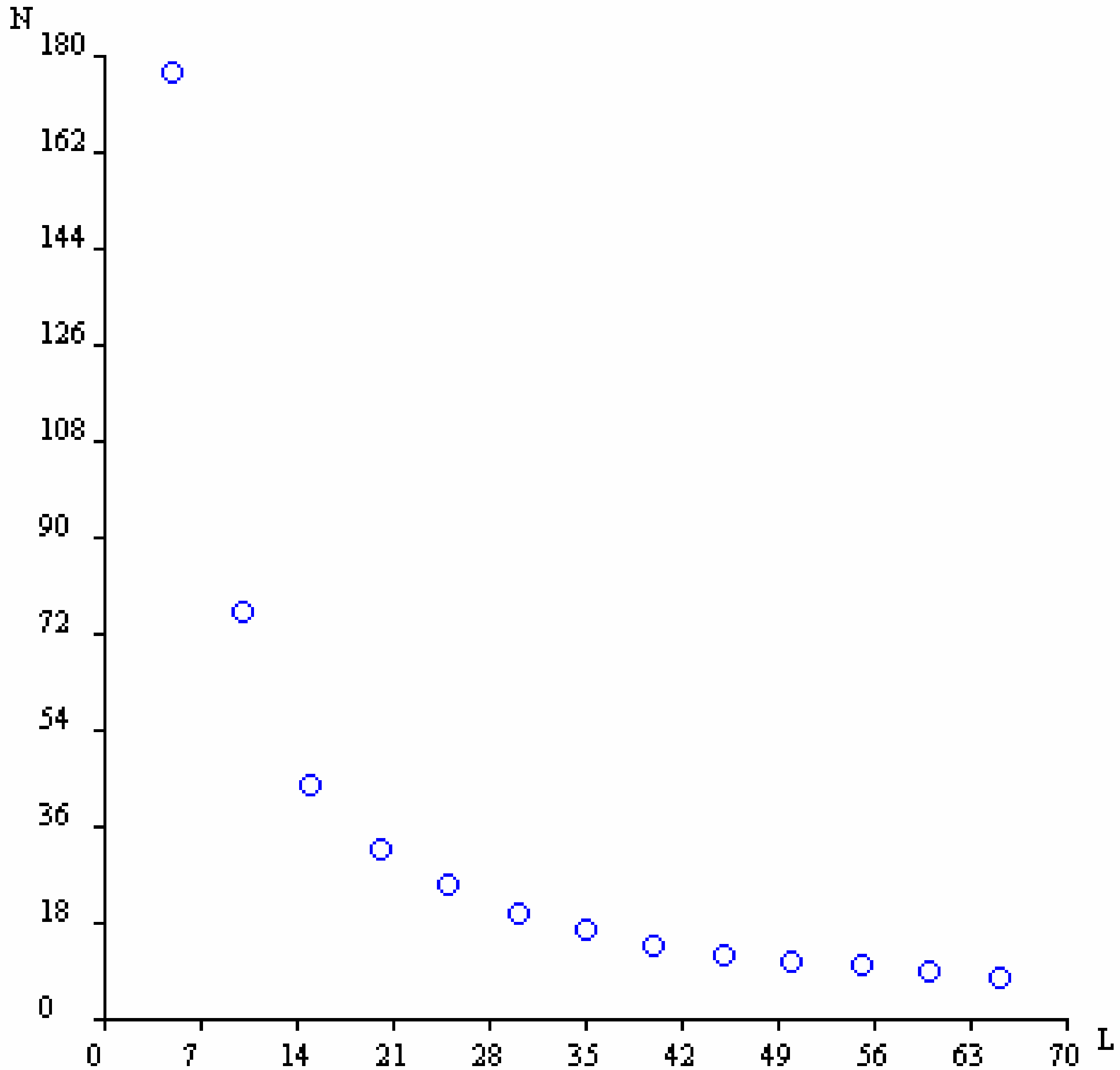
*Length = N(L) * L*



N(50) = 12

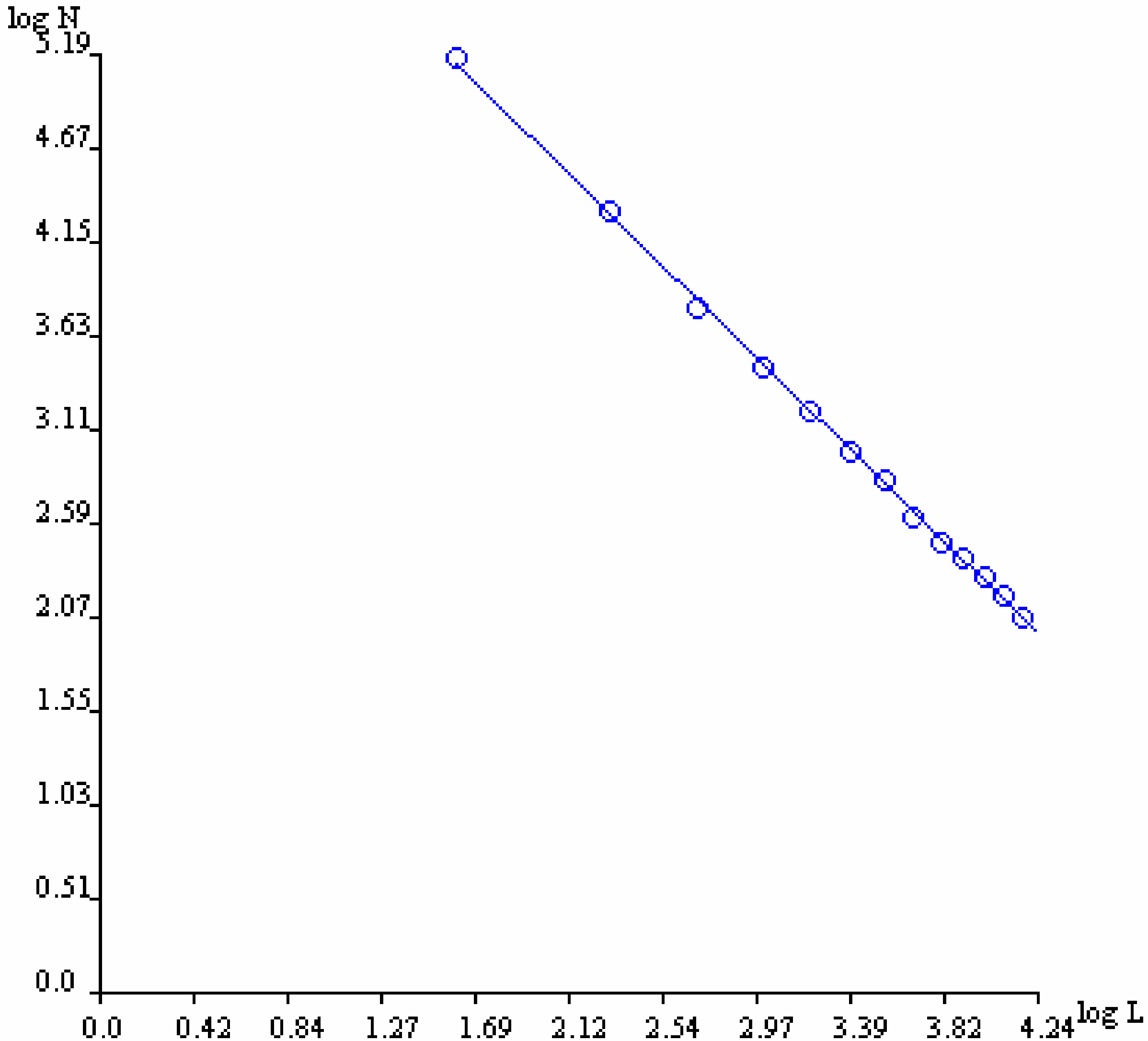


N(25) = 26



L = size of ruler or grid

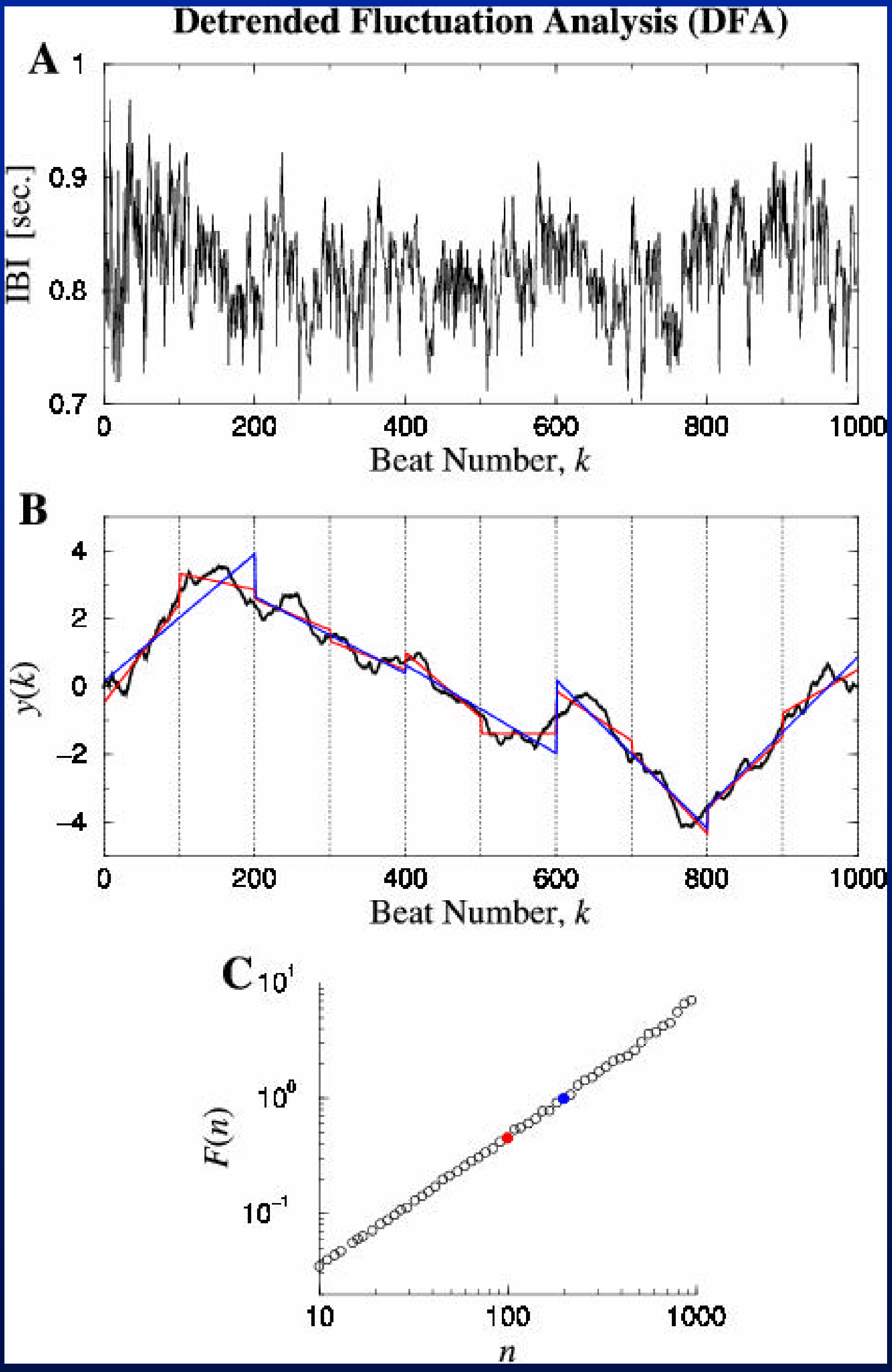
N = number laid down



L = size of ruler or grid
N = number laid down

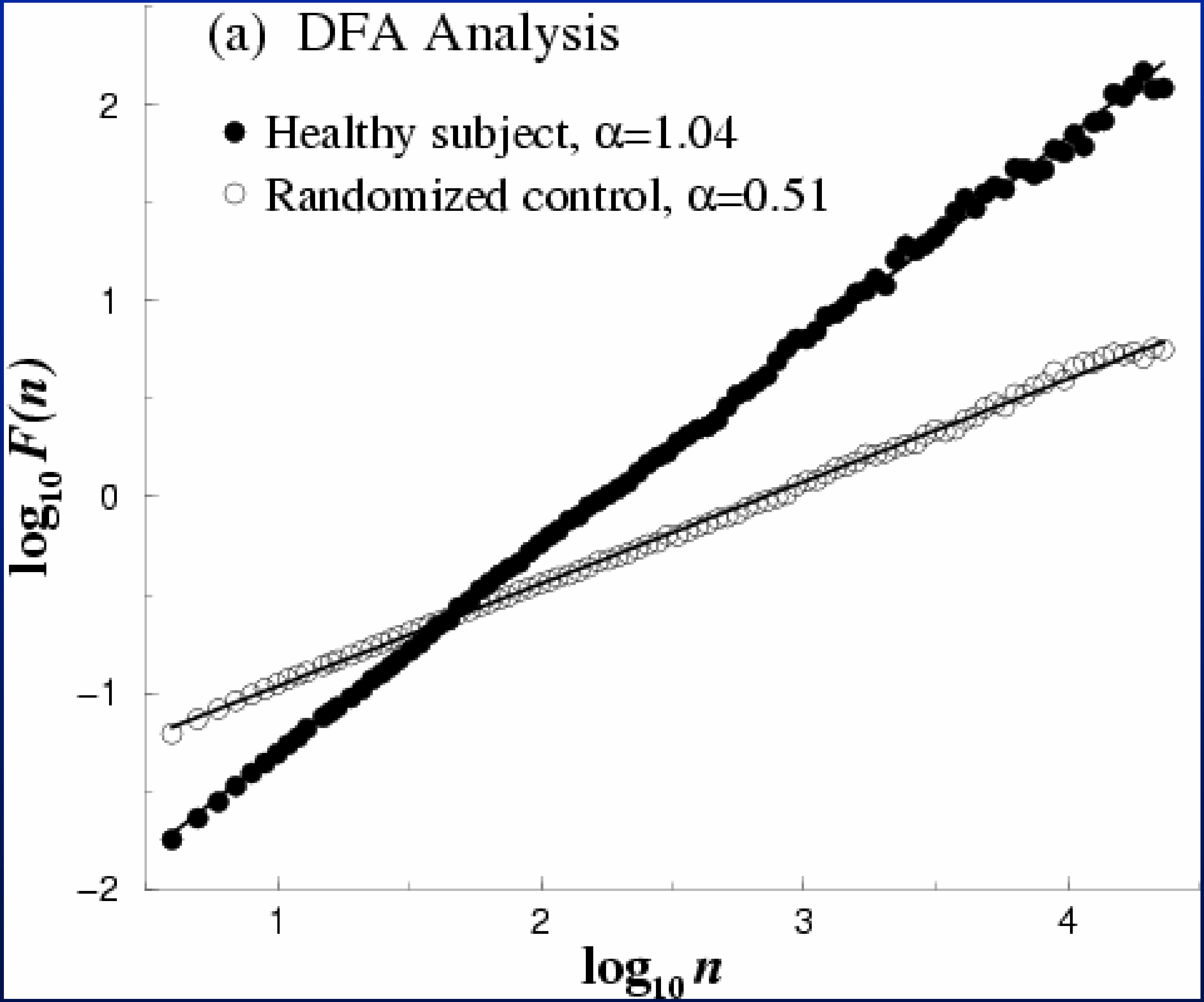
$N = 1180.93L^{-1.19}$

Example 2: Detrended fluctuation analysis (DFA) to measure a self-similarity parameter of fractal processes

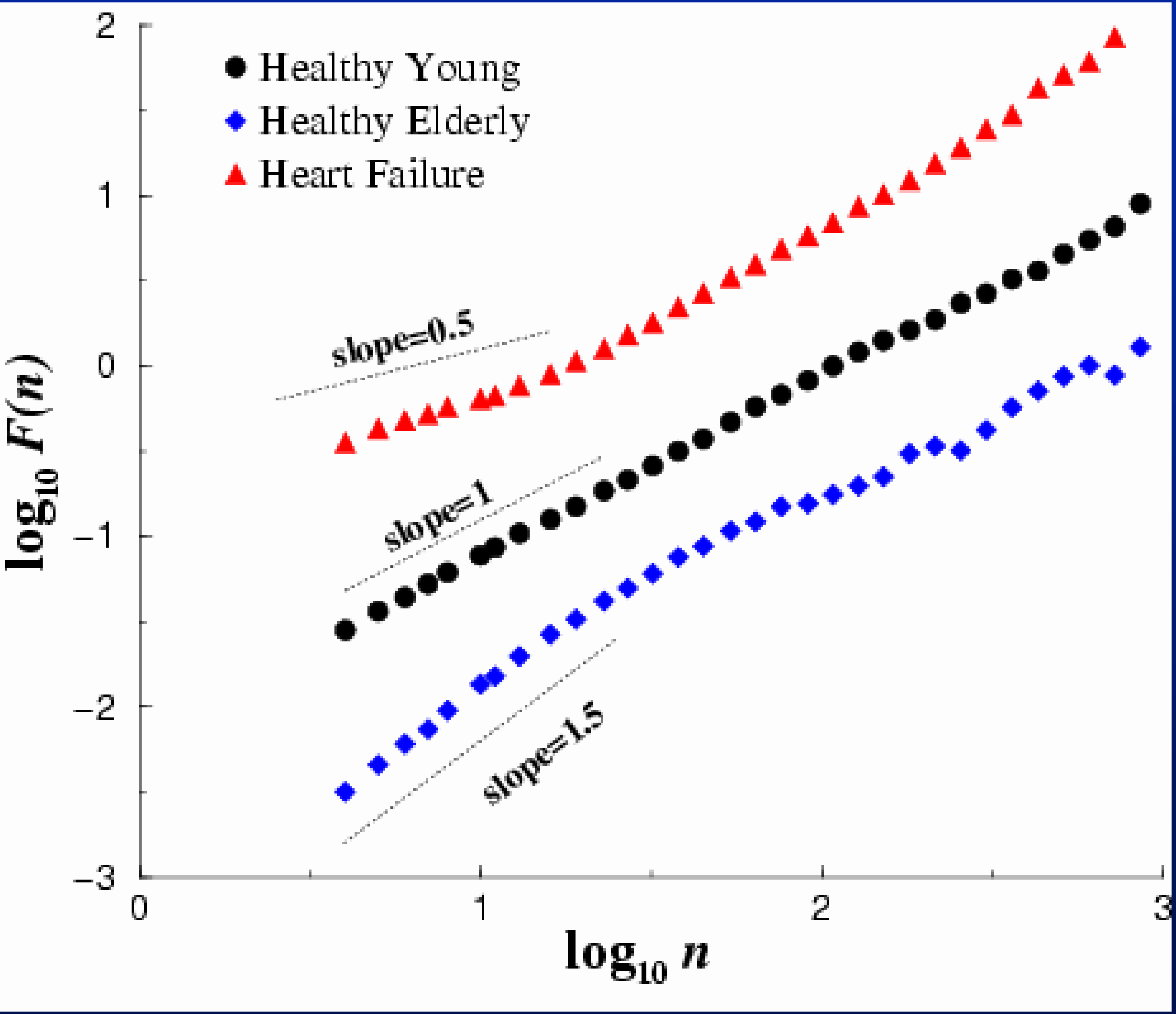


Peng *et al*. Chaos 1995: 5; 82.

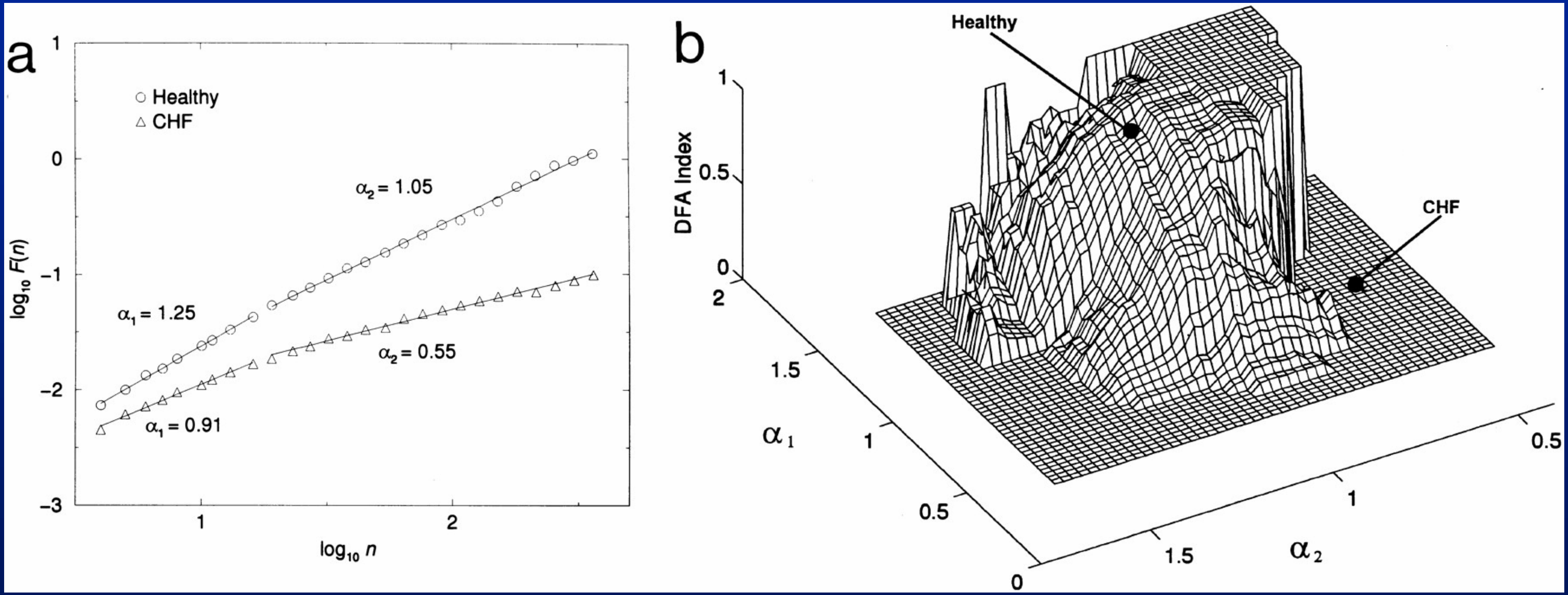
> 500 citations



(a) DFA Analysis
Healthy subject, α=1.04
Randomized control, α=0.51
2
1
0
−1
−2
1
2
3
4
log10 F(n)
log10 n

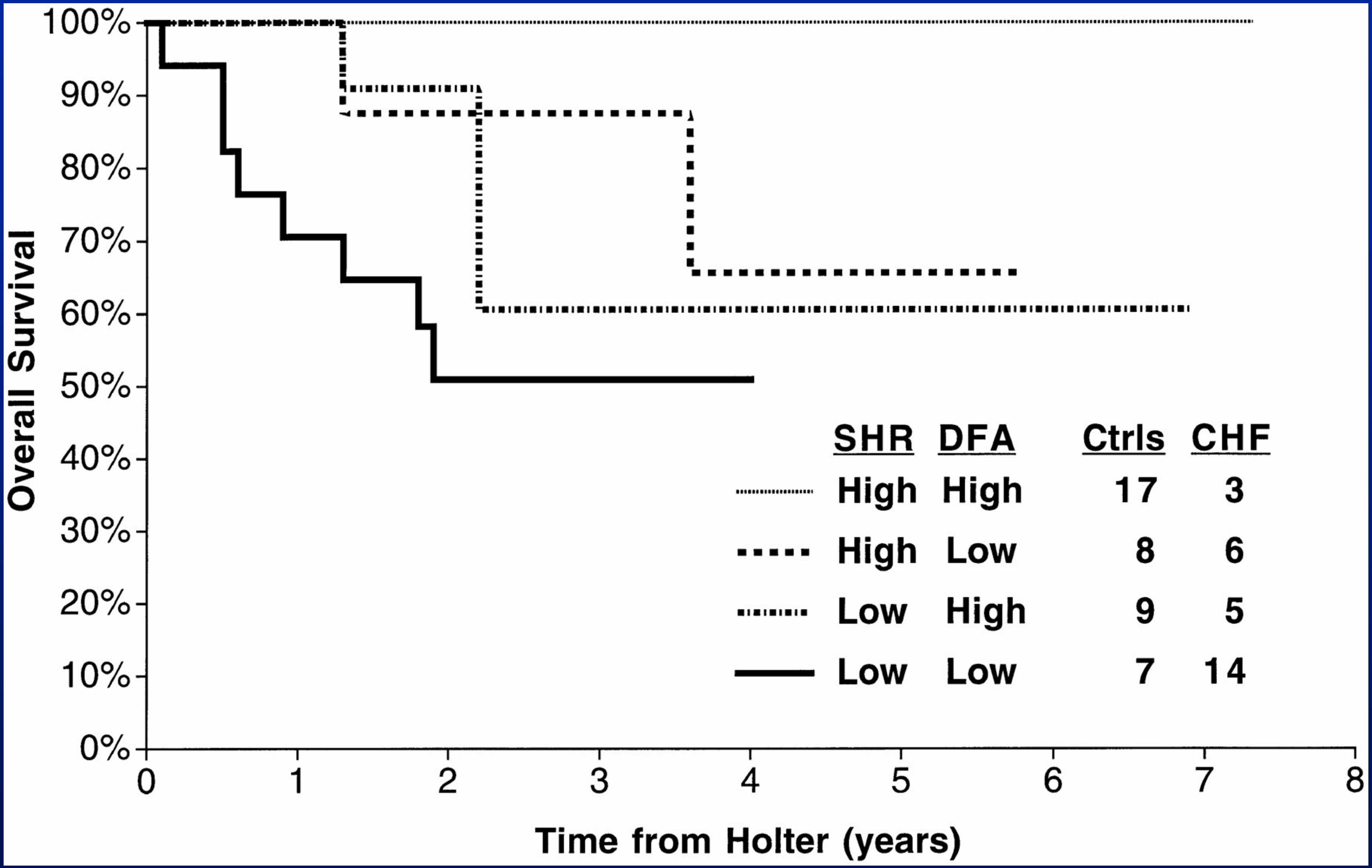

Healthy Young
Healthy Elderly
Heart Failure
slope=0.5
slope=1
slope=1.5
$\log_{10} F(n)$
$\log_{10} n$
2
1
0
−1
−2
−3
0
1
2
3

Application to Framingham Heart Study



Ho, Moody, Peng, Mietus, Larson, Levy, Goldberger.

Circulation 1997; 96: 842-848



Ho, Moody, Peng, Mietus, Larson, Levy, Goldberger.

Circulation 1997; 96: 842-848

Fractal Correlation Properties of R-R Interval Dynamics and Mortality in Patients With Depressed Left Ventricular Function After an Acute Myocardial Infarction

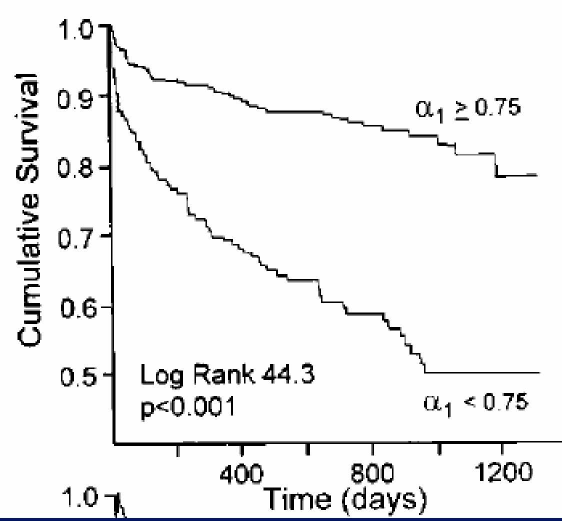
Heikki V. Huikuri, MD; Timo H. Mäkikallio, MD; Chung-Kang Peng, PhD; Ary L. Goldberger, MD; Ulrik Hintze, MD; Mogens Møller, MD; for the DIAMOND Study Group

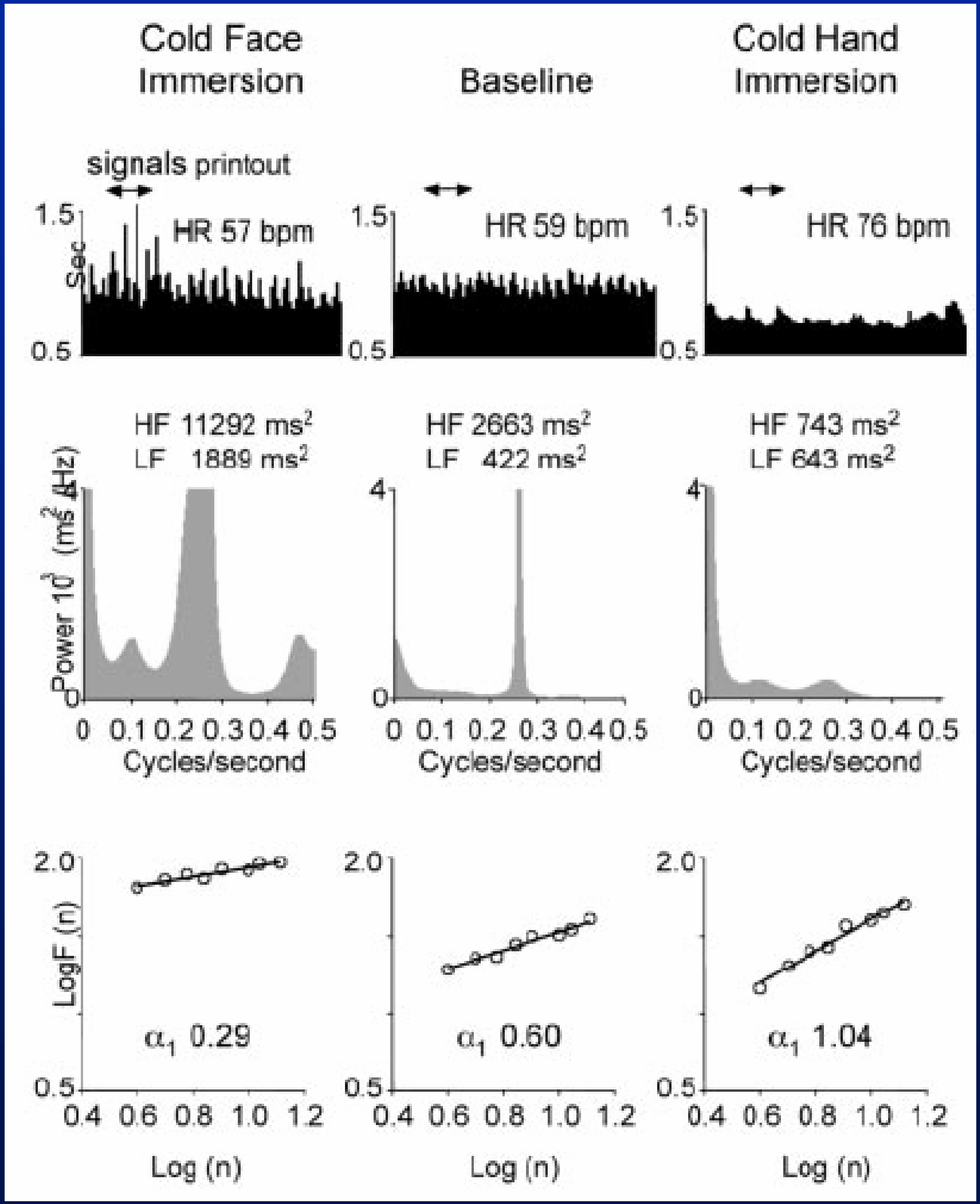
Background—Preliminary data suggest that the analysis of R-R interval variability by fractal analysis methods may provide clinically useful information on patients with heart failure. The purpose of this study was to compare the prognostic power of new fractal and traditional measures of R-R interval variability as predictors of death after acute myocardial infarction.

Methods and Results—Time and frequency domain heart rate (HR) variability measures, along with short- and long-term correlation (fractal) properties of R-R intervals (exponents α_1 and α_2) and power-law scaling of the power spectra (exponent β), were assessed from 24-hour Holter recordings in 446 survivors of acute myocardial infarction with a depressed left ventricular function (ejection fraction ≤35%). During a mean±SD follow-up period of 685±360 days, 114 patients died (25.6%), with 75 deaths classified as arrhythmic (17.0%) and 28 as nonarrhythmic (6.3%) cardiac deaths. Several traditional and fractal measures of R-R interval variability were significant univariate predictors of all-cause mortality. Reduced short-term scaling exponent α_1 was the most powerful R-R interval variability measure as a predictor of all-cause mortality (α_1 <0.75, relative risk 3.0, 95% confidence interval 2.5 to 4.2, $P<0.001$). It remained an independent predictor of death ($P<0.001$) after adjustment for other postinfarction risk markers, such as age, ejection fraction, NYHA class, and medication. Reduced α_1 predicted both arrhythmic death ($P<0.001$) and nonarrhythmic cardiac death ($P<0.001$).

Conclusions—Analysis of the fractal characteristics of short-term R-R interval dynamics yields more powerful prognostic information than the traditional measures of HR variability among patients with depressed left ventricular function after an acute myocardial infarction. (***Circulation.* 2000;101:47-53.**)

Key Words: mortality ■ heart rate ■ infarction





"The fractal organization of human HR dynamics is determined by a delicate interplay between sympathetic and vagal outflow, with the breakdown of fractal HR behavior toward more random dynamics occurring during coactivation of sympathetic and vagal outflow."

Tulppo *et al*. Circulation 2005; 112:314-319

Beyond Fractal: Multifractal Analysis

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A Brief Overview of Multifractal Time Series

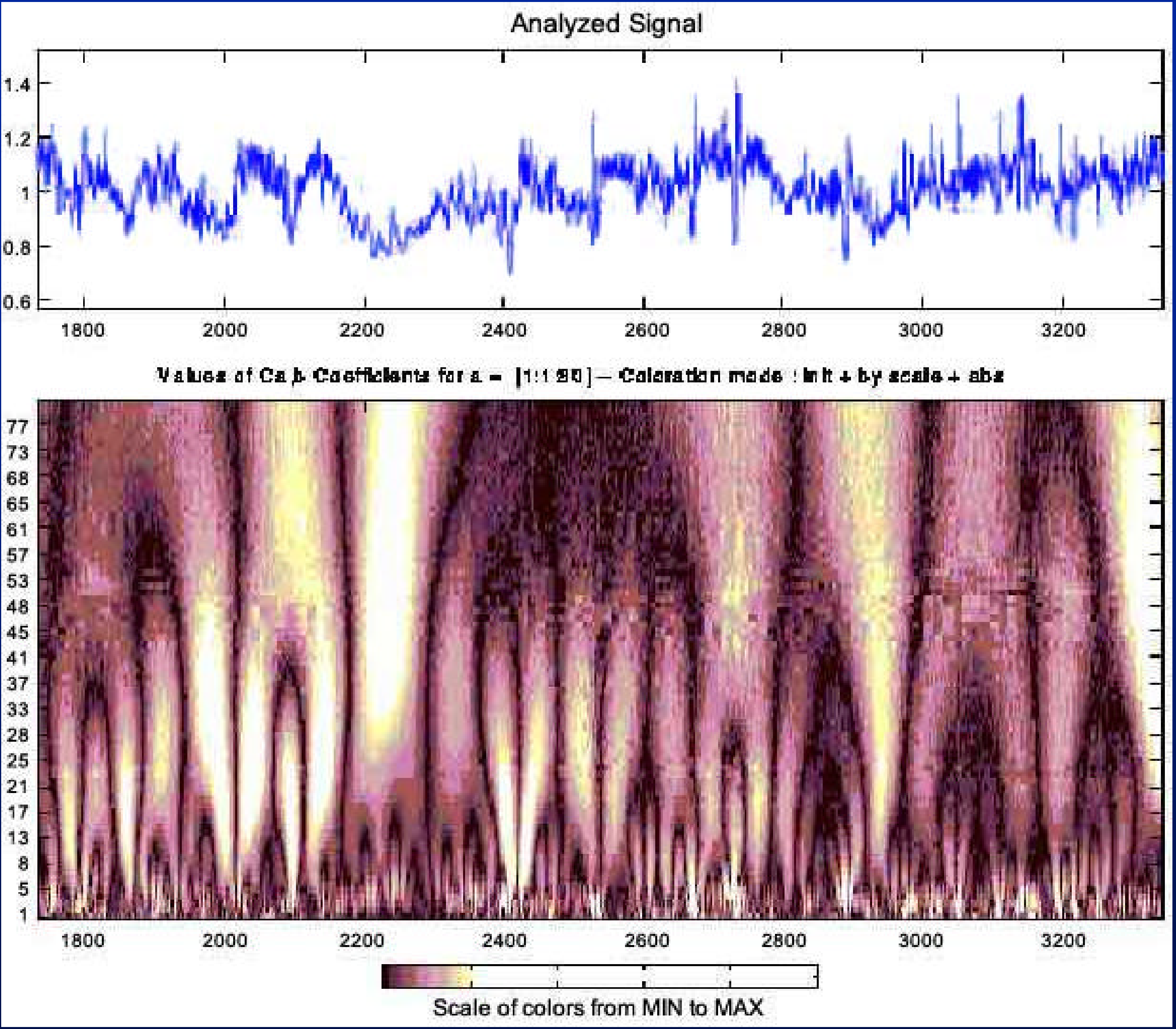
Luís Amaral

In the Exploring Patterns in Nature tutorials, we observed how disordered, irregular, fractal patterns can be quantified in terms of their spatial fractal dimension. Here we study fractal (and multifractal) patterns of a different sort: patterns in time.

This overview attempts to give a short operational review of multifractality in time series. For this reason, formal definitions and derivations are not discussed; see Refs. 1-4 for more in-depth reviews.

- Part 1: Fractal behavior in time series
- Part 2: Using wavelets to detect singular behavior
- Part 3: The fractal dimension of the singular behavior
- Part 4: The singularity spectra of multifractal signals
- Part 5: What one learns from the singularity spectra of multifractal signals
- Part 6: Multifractality of healthy human heart rate
- Bibliography

Software for multifractal analysis of time series is available here.


Analyzed Signal
1.4
1.2
1
0.8
0.6
1800
2000
2200
2400
2600
2800
3000
3200
Values of Ca,b Coefficients for a = [1:1:80] – Coloration mode : init + by scale + abs
77
73
68
65
61
57
53
48
45
41
37
33
28
25
21
17
13
8
5
1
1800
2000
2200
2400
2600
2800
3000
3200
Scale of colors from MIN to MAX

Conclusions

- Heart rate time series exhibit fractal (self-similar) properties
- We can quantify the exponents of fractal scaling
- These exponents are altered in disease and aging