What is Nonlinear Dynamics?

HRV 2006: Techniques, Applications, and New Directions

Daniel Kaplan

Macalester College

Saint Paul, Minnesota

Dynamics and Complexity

- Historically
  - Simple systems generate simple behavior
    - e.g. the pendulum
    - orbits of planets in the solar system
  - Complex systems generate complex behavior
    - e.g. the economy
  - In between is in between
    - e.g., NMR spectroscopy

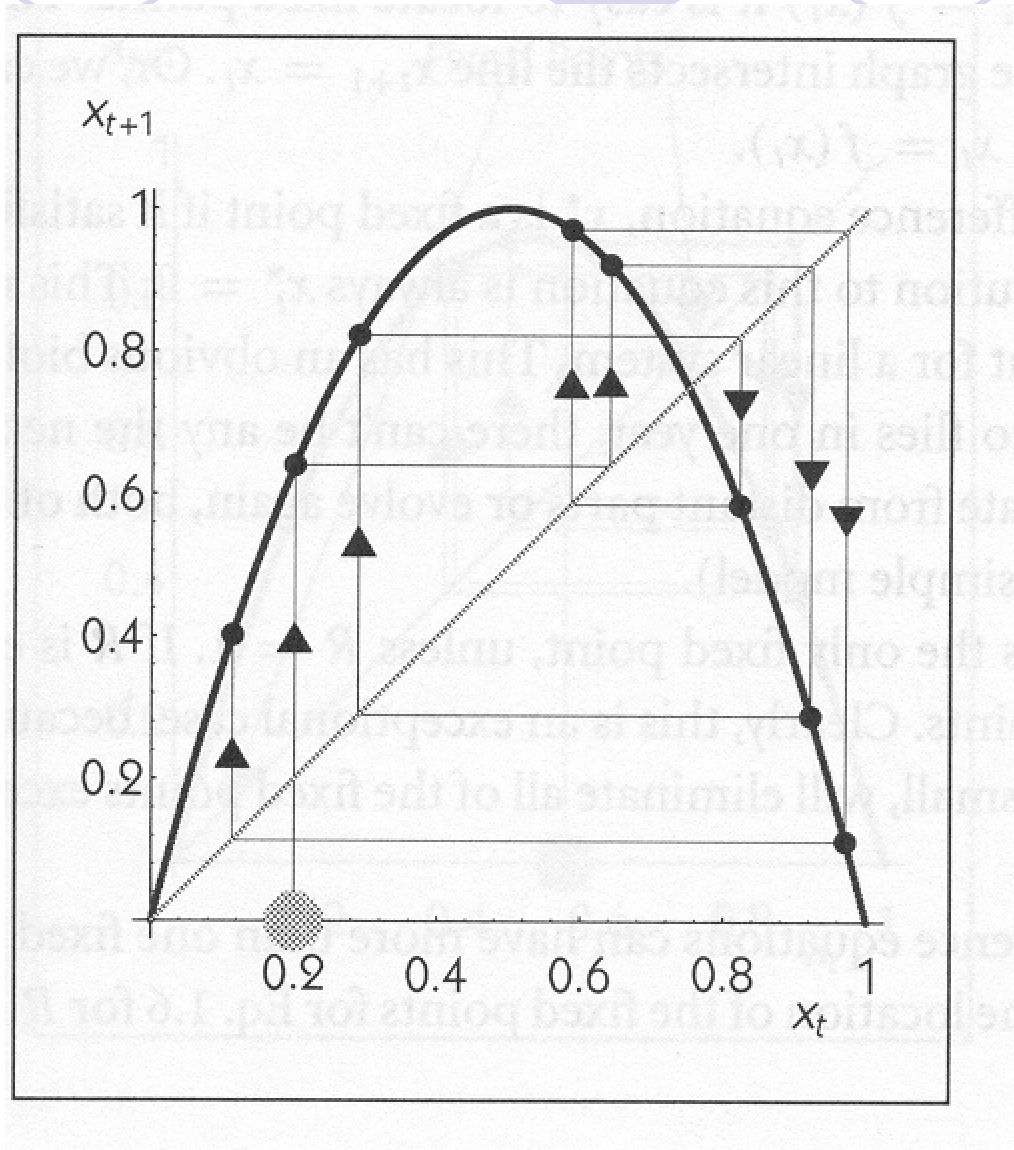
Dynamics since the 1970s

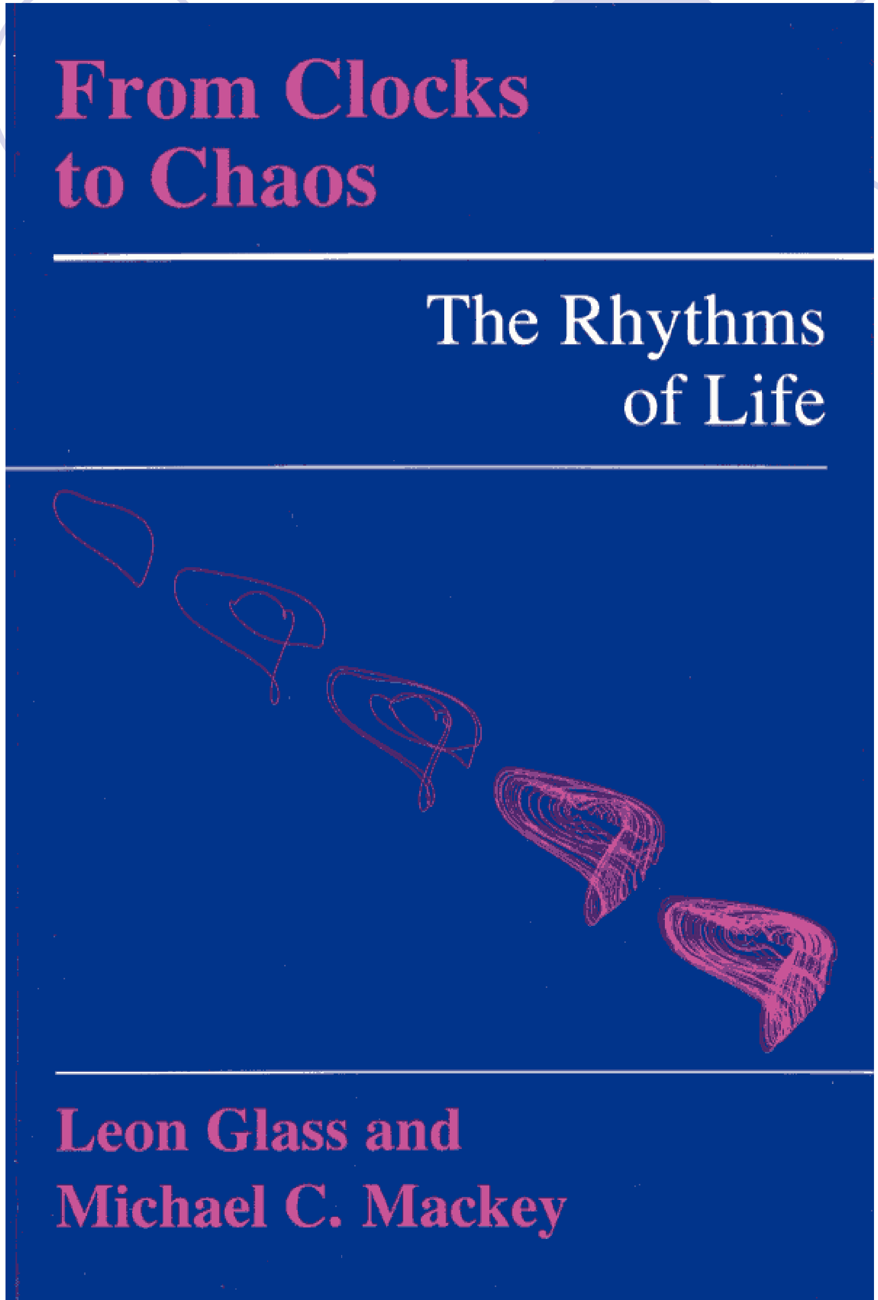
- Simple systems can generate complex-looking behavior.
  - Chaos
  - Planets are not so simple after all, asteroids
- Complex systems can generate simple behavior
  - Fixed points, limit cycles
- New types of behavior can *emerge* when coupling systems
  - Spiral waves in tachycardia and fibrillation.

Components of Dynamics

- A **state** that can change in time.
  - a sufficient description of the system
- A **rule**/mechanism/process that describes what the new state will be given any existing state.
  - Discrete time: finite-difference equations
  - Continuous time: differential equations
- Possibly a set of **inputs** that connect the system to another one: *coupling*
  - The inputs might be considered random, giving a *stochastic dynamical system*.

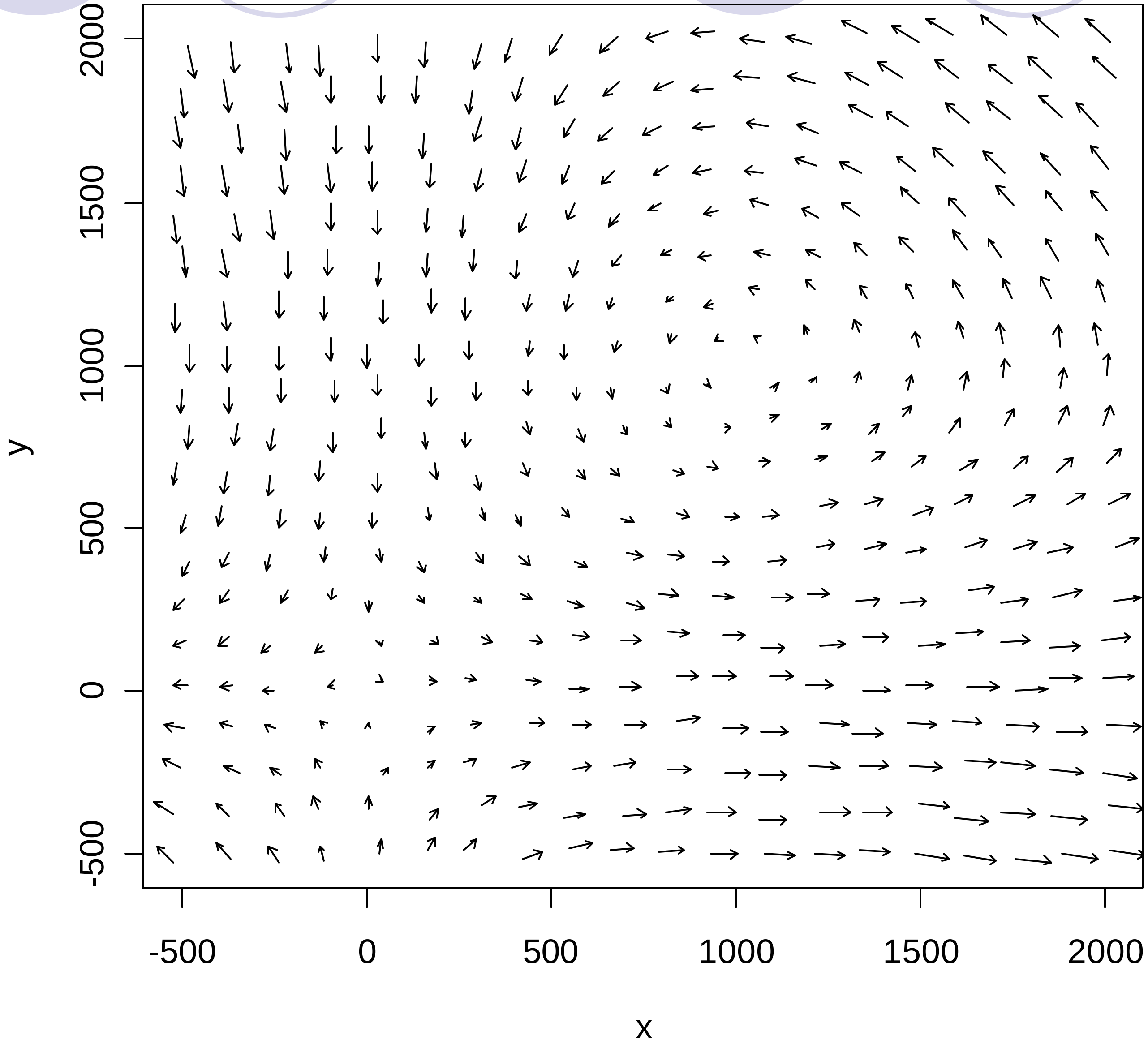
A One-dimensional State





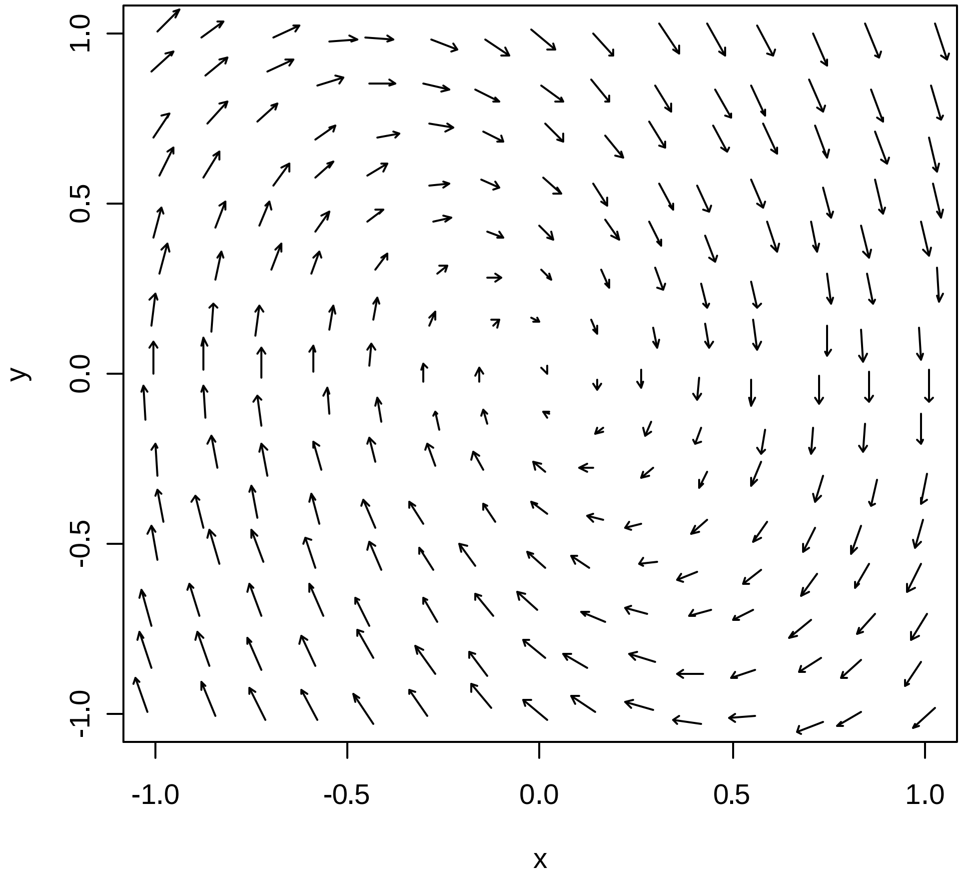
Princeton Univ. Press
1988
0-691-08496-3

A Two-dimensional State



What is Linear Dynamics?

- The dynamical rule is a proportional function of the state.
- Basic phenomena
  - Exponential growth, exponential decay
  - Sine-wave oscillation
  - Modulated sine-wave oscillation
  - Combinations of the above
- Stability: decays to steady state or grows to infinity. There’s no in-between behavior



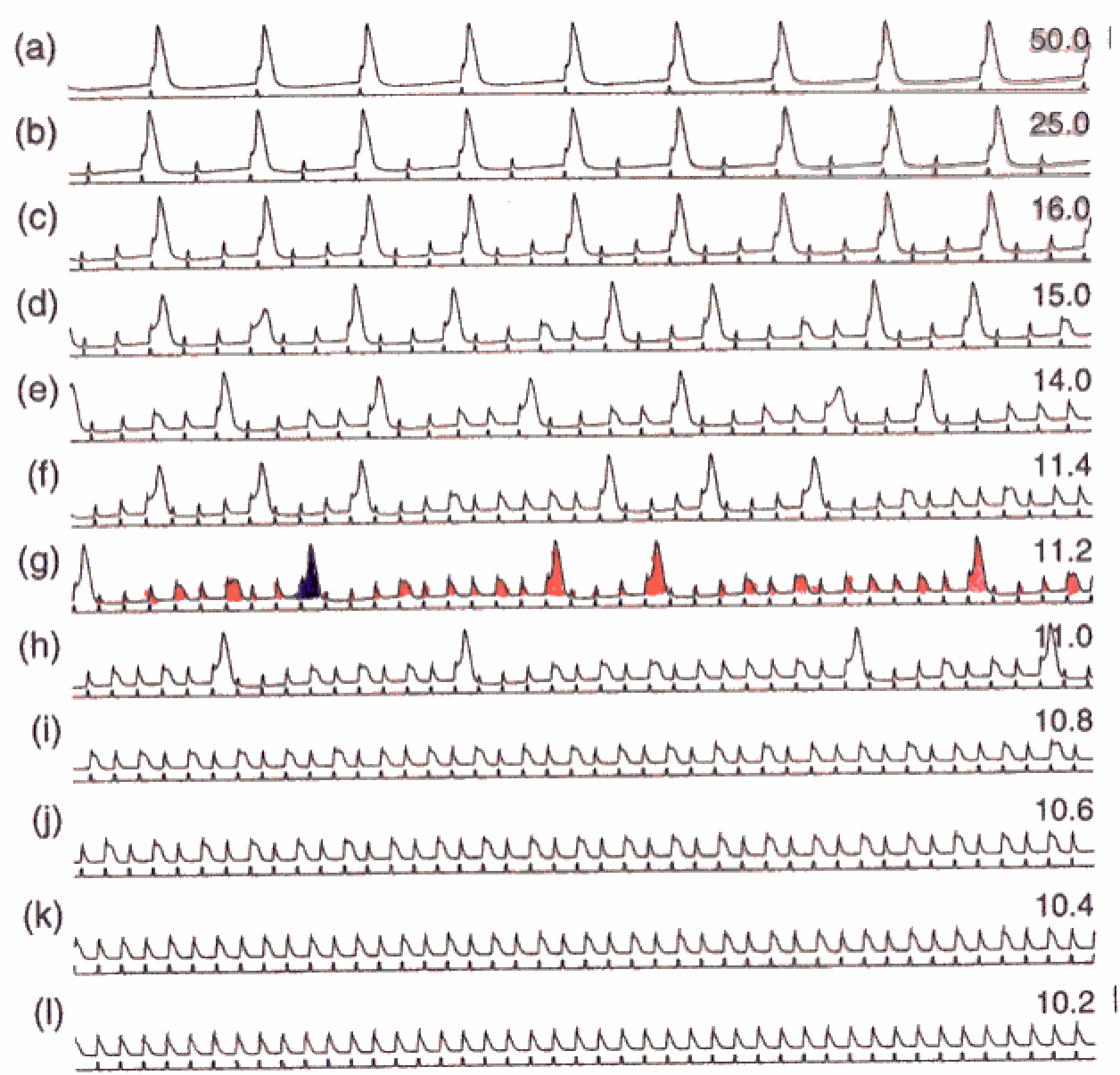
1.0
0.5
0.0
-0.5
-1.0
y
-1.0
-0.5
0.0
0.5
1.0
x

Linear Dynamics with an Input

- Proportional response: doubling the input leads to doubling the output.
- Sine-wave input gives a sine-wave output.
  - Respiration and RSA in HRV
- Superposition and decomposition:
  - Break input into components, find response to each component, then add together to find the overall response.
  - Fourier decomposition.

Ways that Linear Models are Wrong for HRV

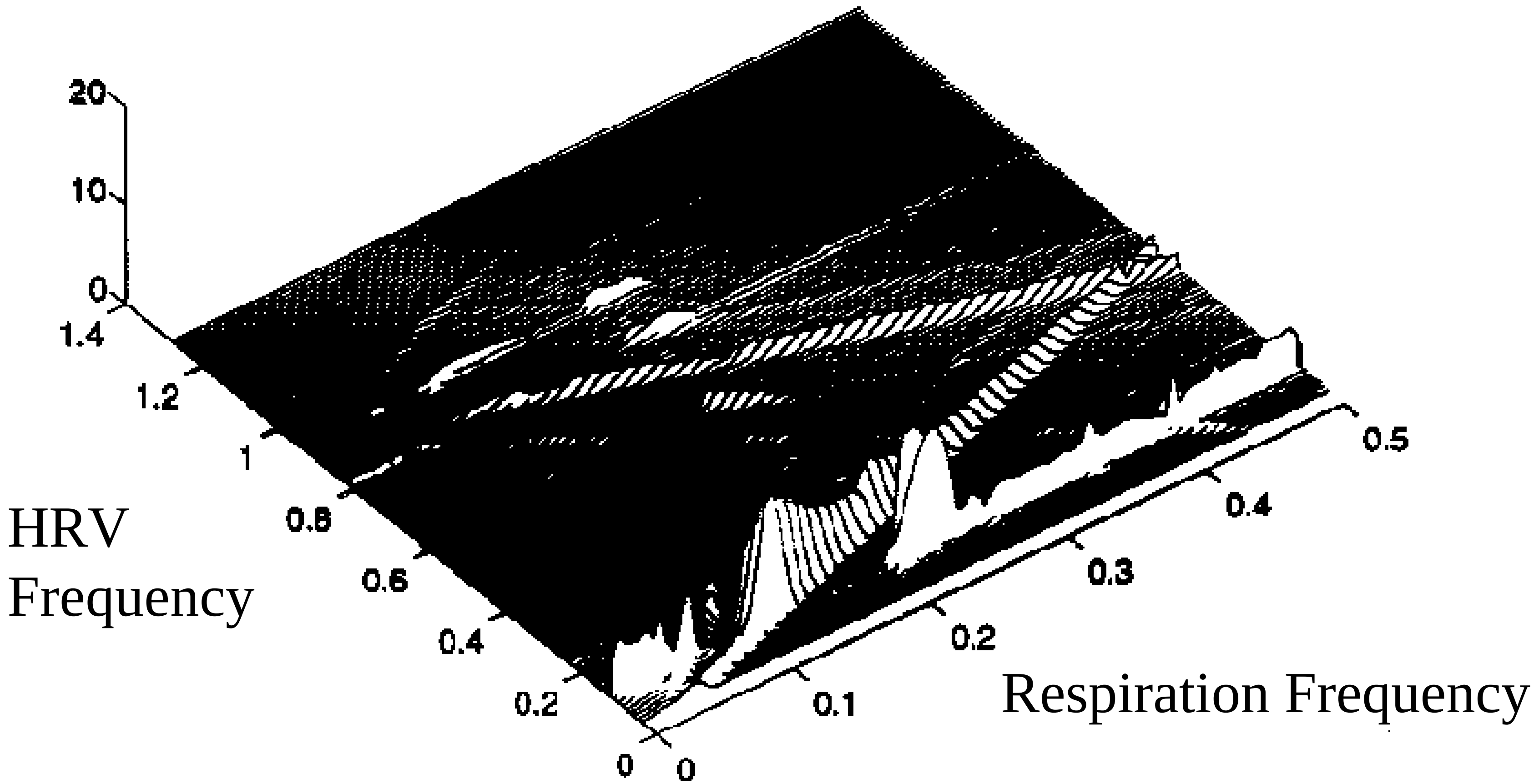
- Proportional response?
  - Limits to ventricular response to pacing. If the response were proportional, the heart rate would be unlimited. [See Leon Glass's session on complex arrhythmogenesis.]


(a)
50.0
(b)
25.0
(c)
16.0
(d)
15.0
(e)
14.0
(f)
11.4
(g)
11.2
(h)
11.0
(i)
10.8
(j)
10.6
(k)
10.4
(l)
10.2

Phenomena Linear Models Can't Capture

- Stability and a Single point attractor?
  - The same cardiac system can have utterly different types of sustained behavior depending on the initial state, e.g. NSR, tachycardia, fibrillation.
- Entrainment
- Frequency Pulling

Nonlinear Interactions of HRV Oscillators: Model w/strong 10 second rhythm



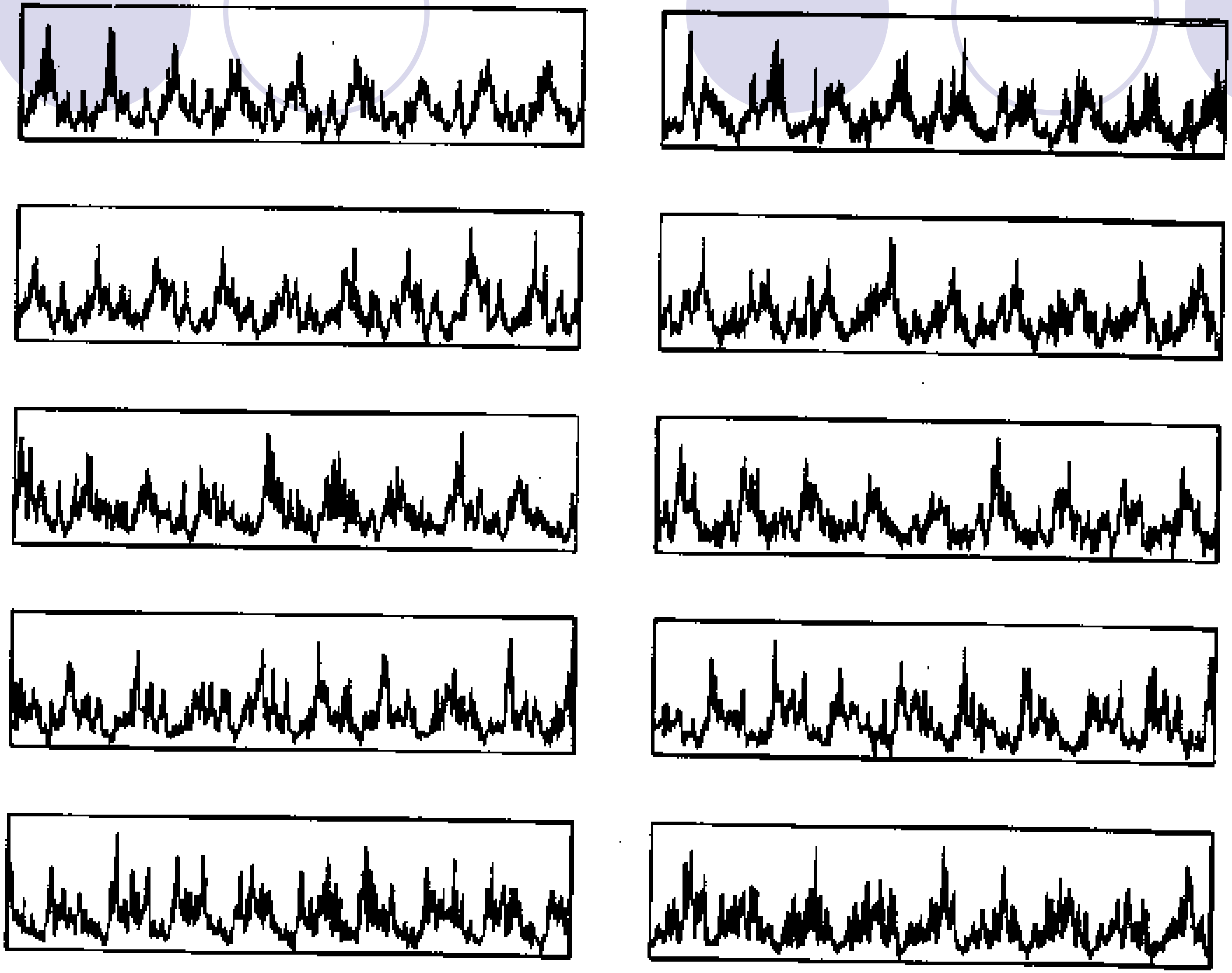
“*All models are wrong, some models are useful*.” –George Box

- In some situations, linear models might be more useful than nonlinear ones.
- State might be so complicated that we can’t construct any detailed proxy for it, so the dynamics are effectively random.
- Outside inputs may dominate internal dynamics.
  - e.g. random-interval breathing
- Nonstationarity
  - is there enough repetition to see the dynamical rule?
  - is the state changing according to a constant dynamical rule or is the rule changing in time?
- Parsimony. Linear models can capture behavior with few parameters. Short data sets, changing dynamics limit our ability to see nonlinearity.

Can We Reject Linear Dynamics? The Surrogate Data Technique

- Surrogate data are random data that are consistent with a linear model that matches the data.
  - Generated with fourier synthesis.
- Can you distinguish between the surrogate data and the actual data?
- Surrogate data can be made stationary, by design.

Surrogates as a Sanity Check



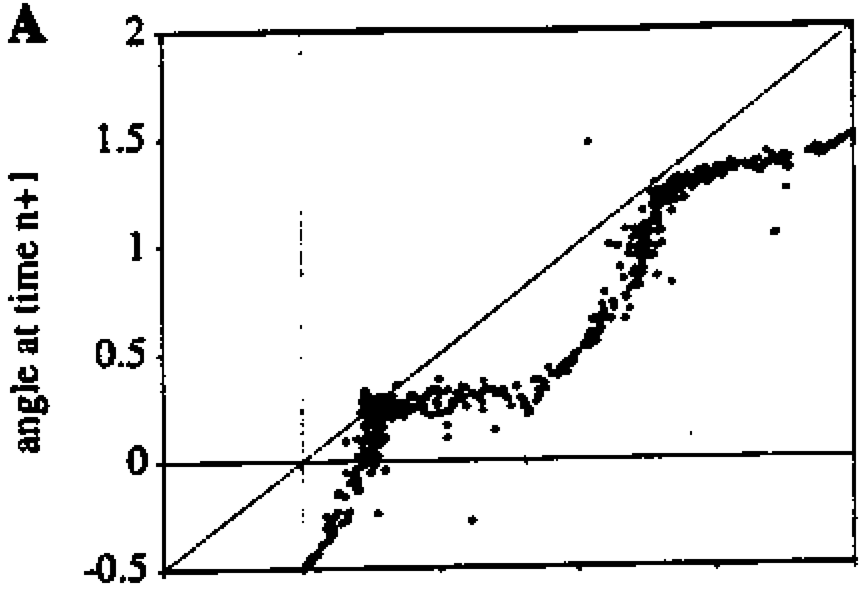
Acth rhythms: Chaos or circadian?

HR/Respiration Coupling

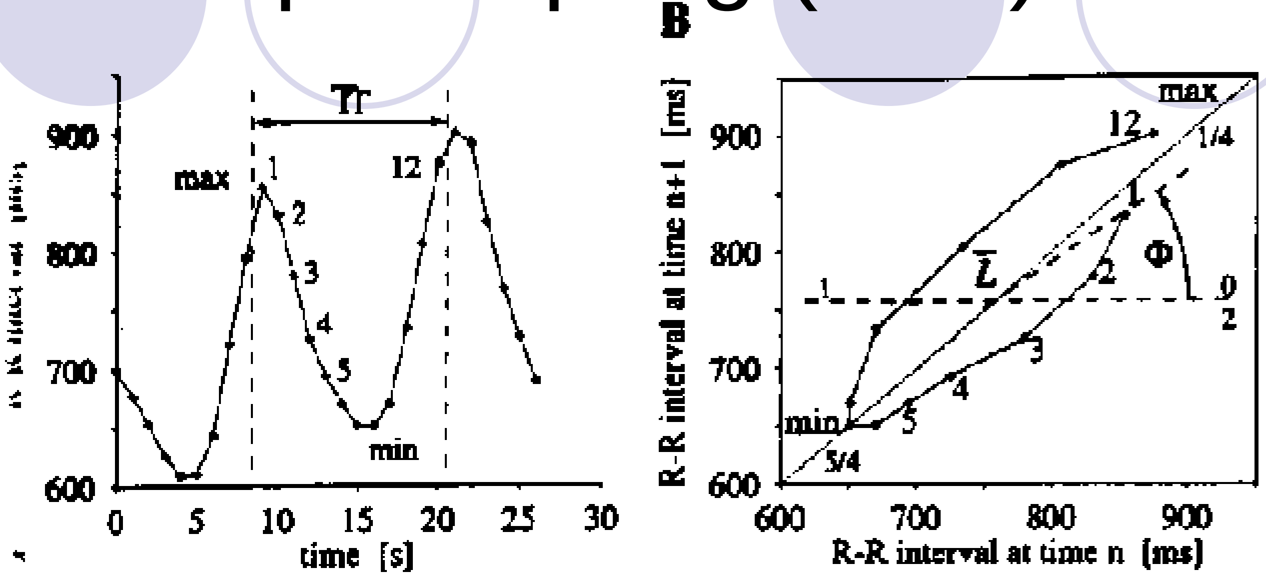
One-dimensional, nonlinear determinism characterizes heart rate pattern during paced respiration

KATRIN SUDER,[1] FRIEDHELM R. DREPPER,[1,2] MICHAEL SCHIEK,[1,2] AND HANS-HENNING ABEL[3]
[1]*Institut für Biologische Informationsverarbeitung and* [2]*Zentrallabor für Elektronik, Forschungszentrum Jülich, 52425 Jülich; and* [3]*Abteilung Kardioanästhesie, Städtisches Klinikum, 38302 Braunschweig, Germany*

A



HR/Resp Coupling (cont)



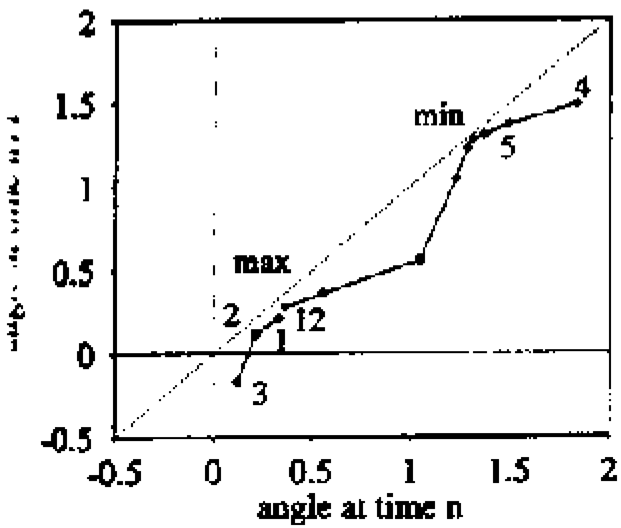
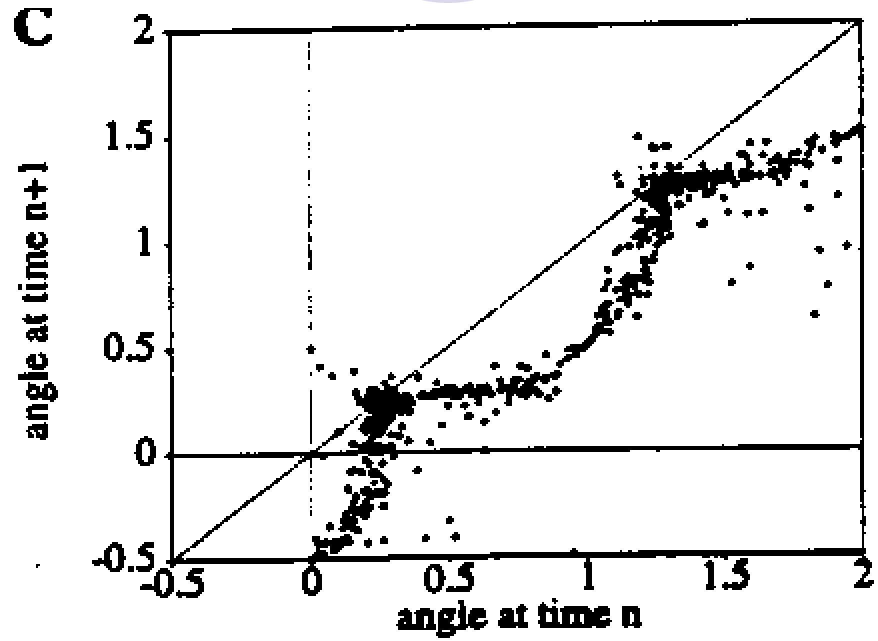


Fig. 3. Graphic explanation of embedding of rotation angles. R-R intervals $L_1, \ldots, L_{12}$ occurring during 1 respiratory cycle Tr (*A*) are embedded two-dimensionally, leading to 12 pairs of successive R-R intervals (L_1, L_2), ..., (L_{12}, L_{13}) (*B*). Rotation angles (Φ) are defined around a center point $(\bar{L}, \bar{L})$. Maximal heartbeats are close to Φ = 1/4, and minimal heartbeats are close to Φ = 5/4. To get a continuous graph of the circle map (*C*), we shifted all ordinates (angles at time *n* + 1) larger than 1.5 by −2.0. Because of the periodicity of angles Φ, this does not change the dynamics, but it eases visual analysis of all circle maps.

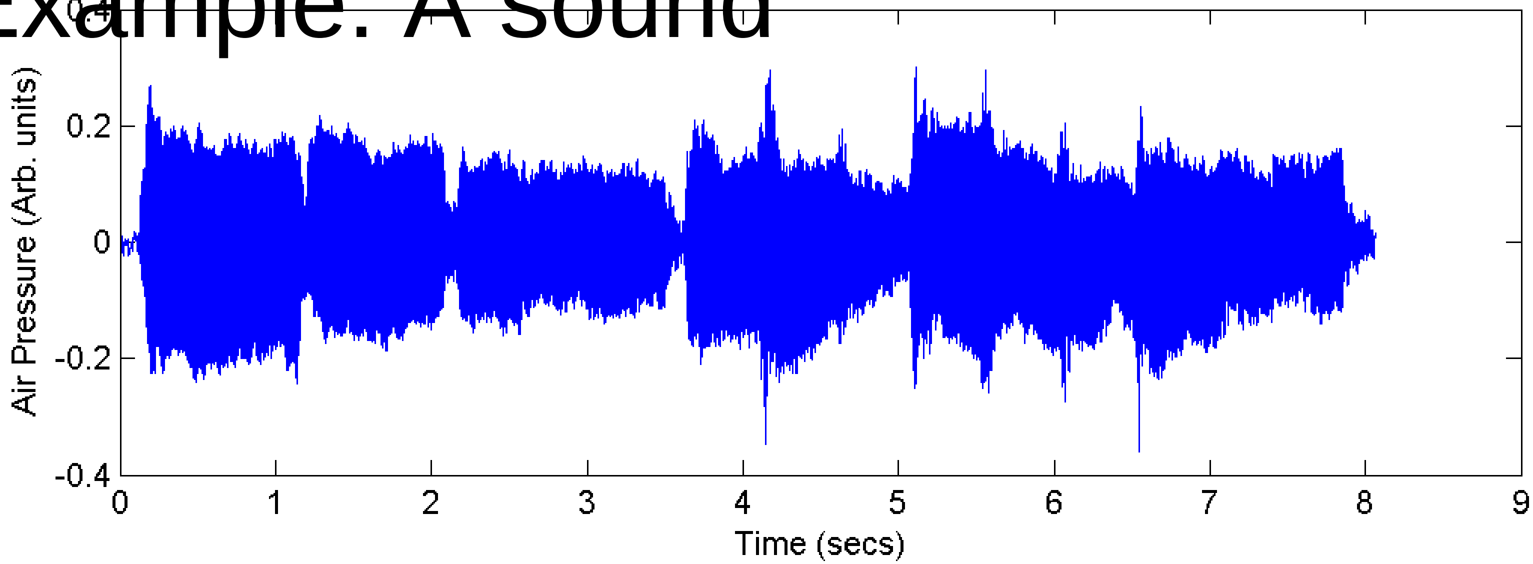
HR/Resp Coupling w/Surrogates

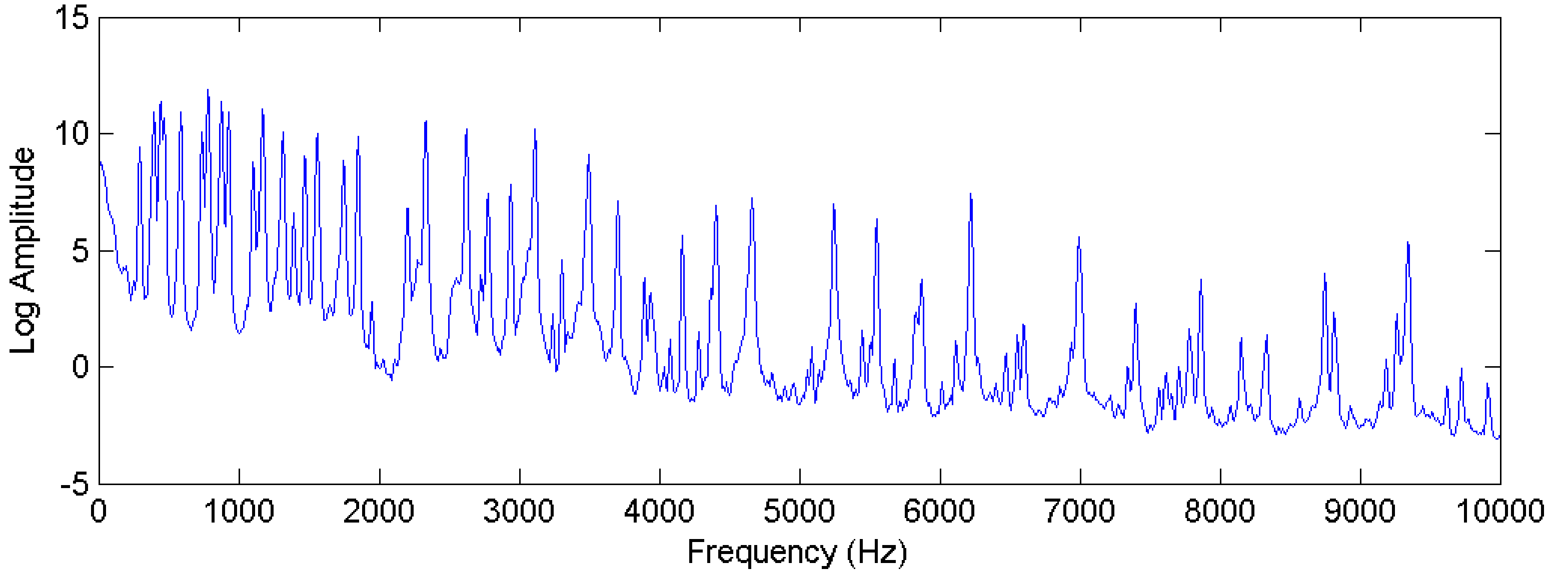
Shuffled Surrogates



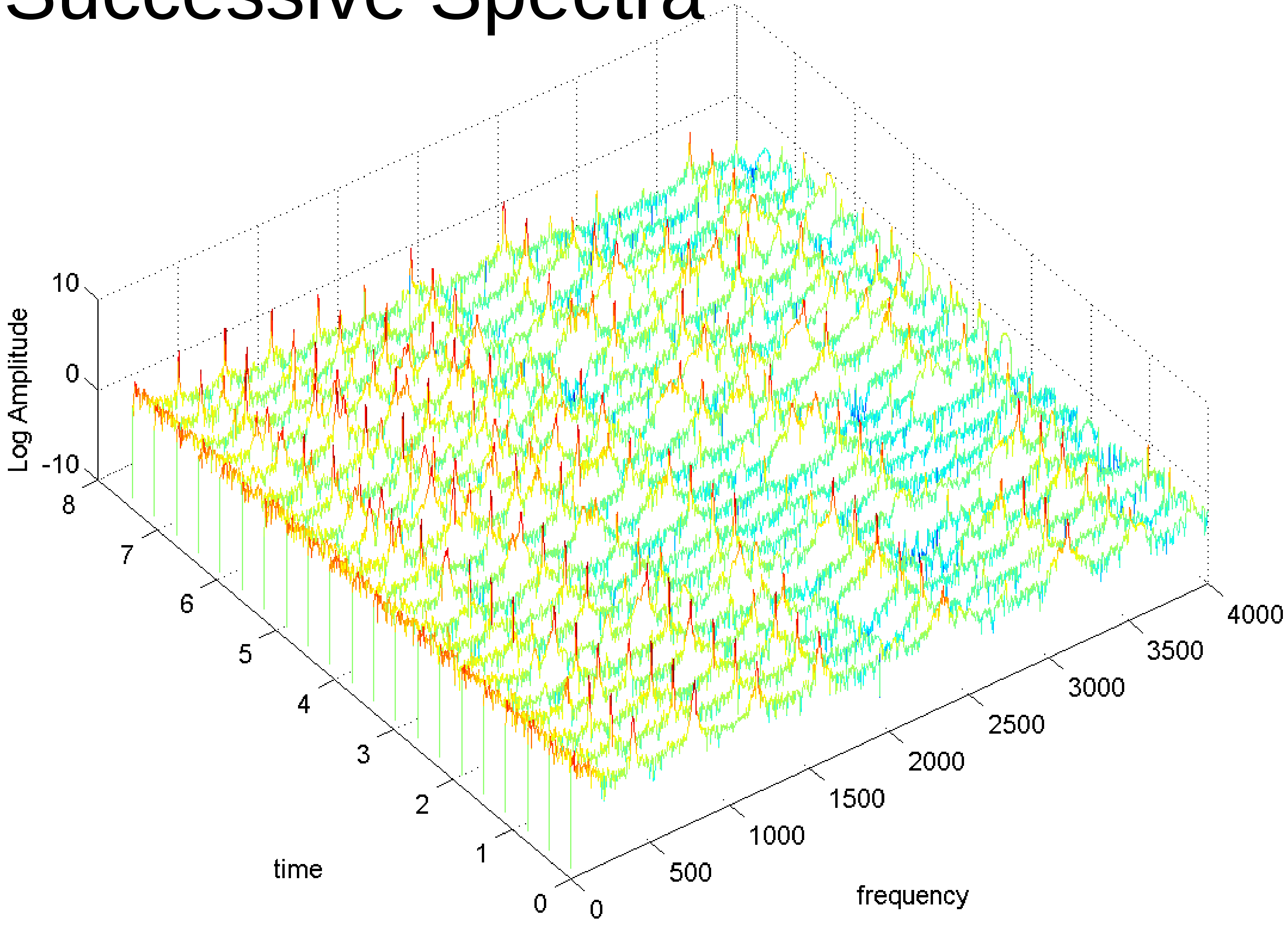
Phase Surrogates

Example: A sound

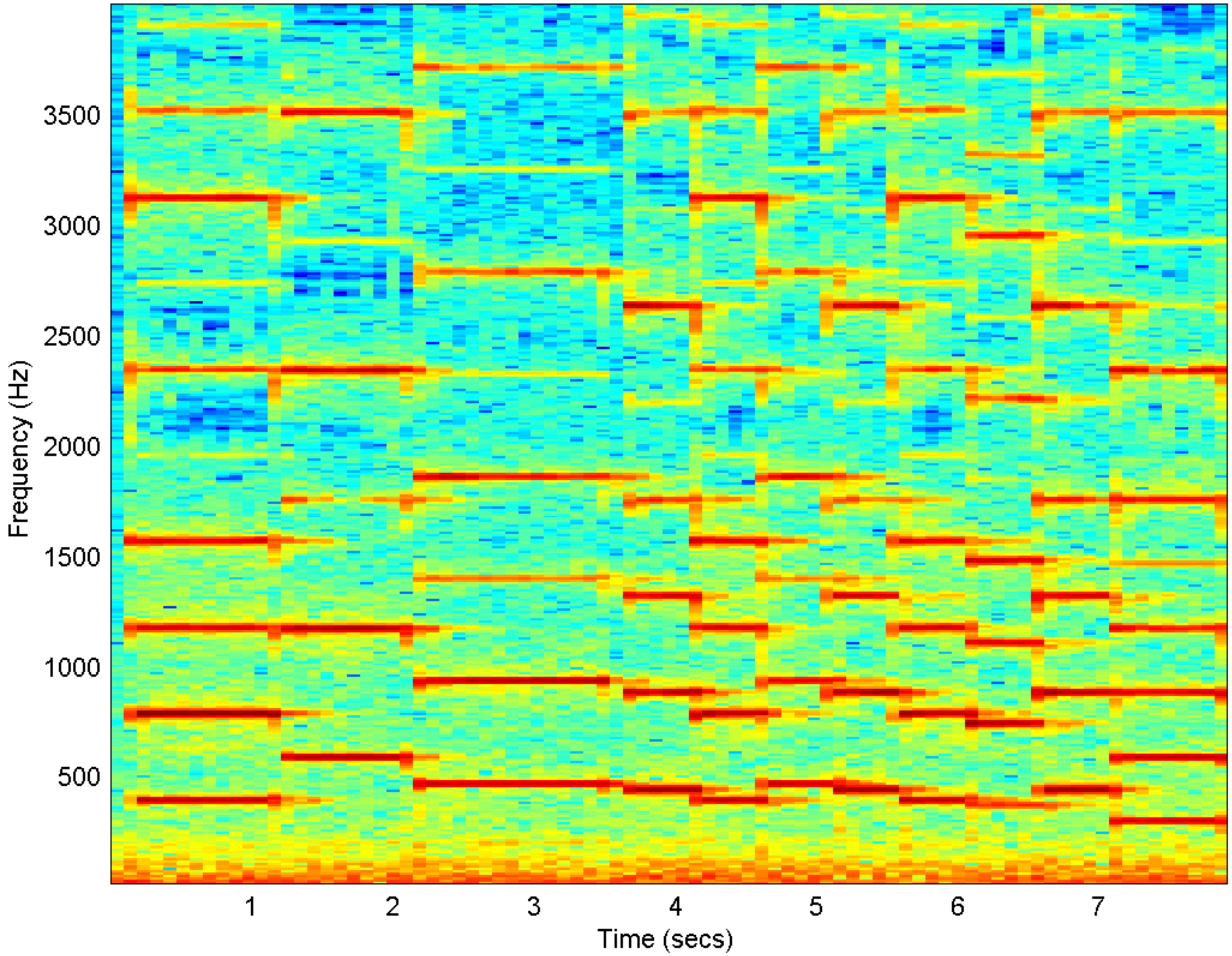




Successive Spectra



The Spectrogram



surrogate

Some Approaches to Nonlinear Dynamics

- Constructing a proxy state: lag embedding
- Example: signal

…, 73, 68, 62, 57, 61, 65, 74, 83, 88, …

becomes a sequence of vectors

(65, 57, 68)

State transition

(74, 61, 62)

State transition

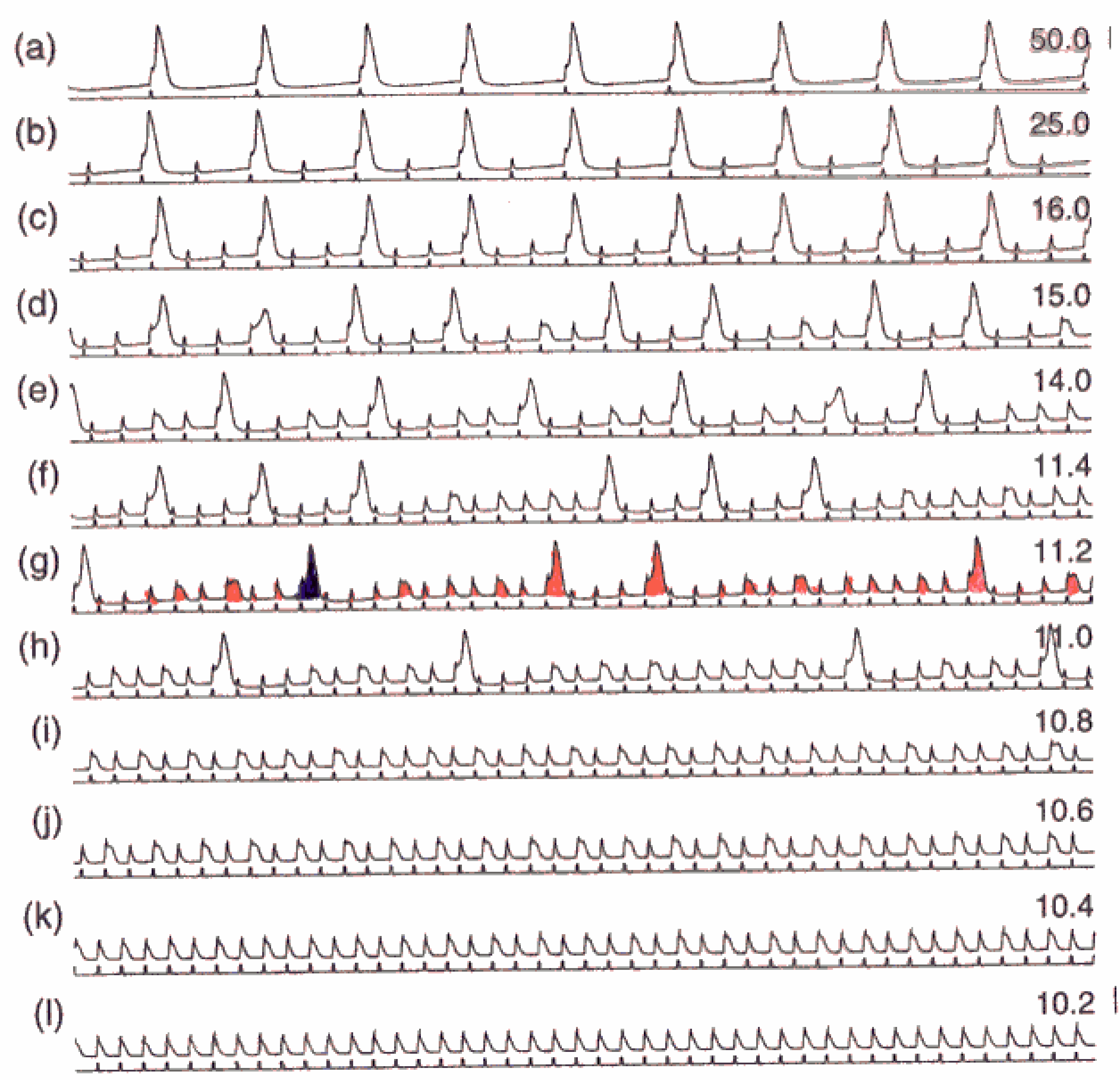
(83, 65, 57)

State transition

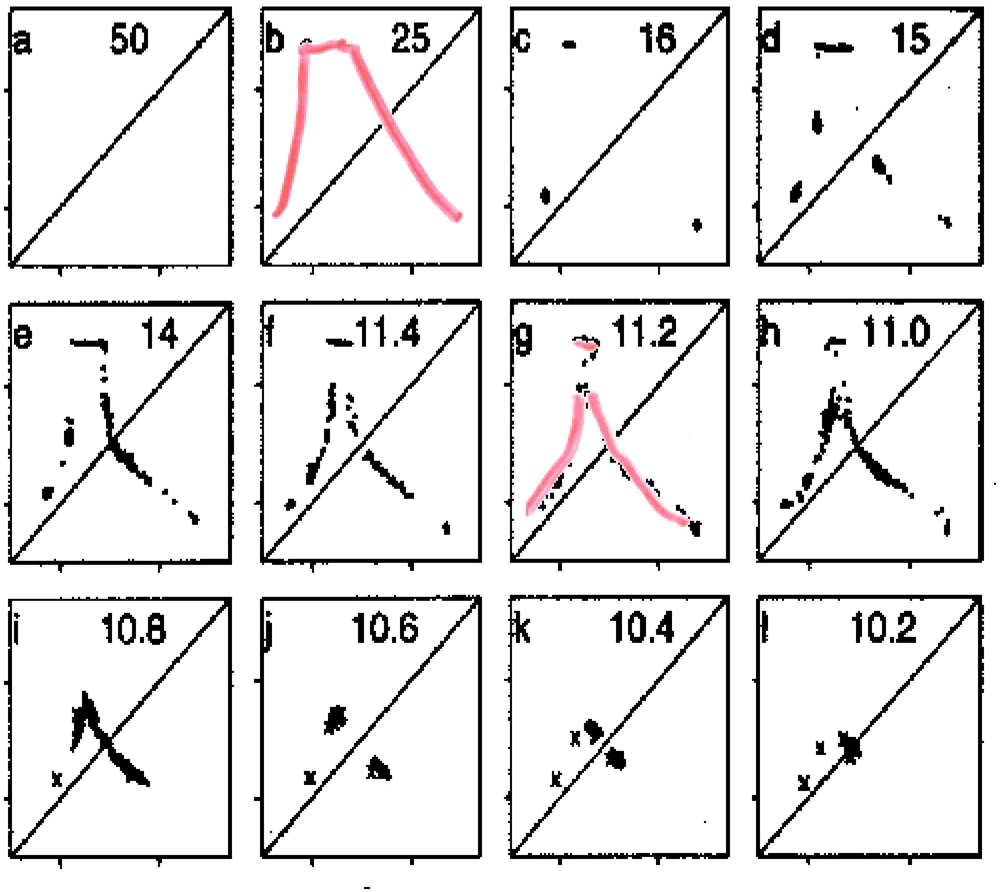
(88, 74, 61)

Model Construction

- Embed time series
- Infer dynamical rule from present->future state-transition pairs
  - Many model architectures possible:
- Nonlinear Prediction
  - How well does the model predict the time series itself?


(a)
50.0
(b)
25.0
(c)
16.0
(d)
15.0
(e)
14.0
(f)
11.4
(g)
11.2
(h)
11.0
(i)
10.8
(j)
10.6
(k)
10.4
(l)
10.2

Squid Return Maps



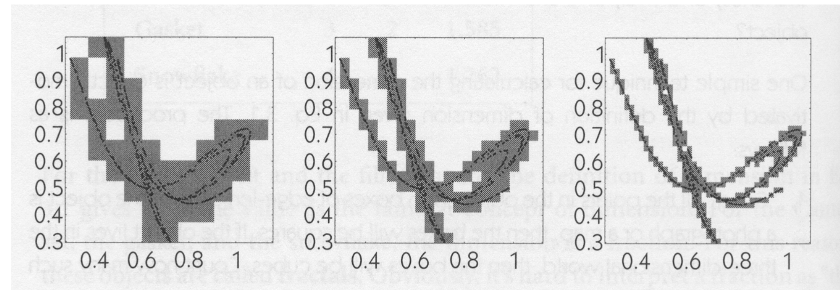
Entropy and Dimension

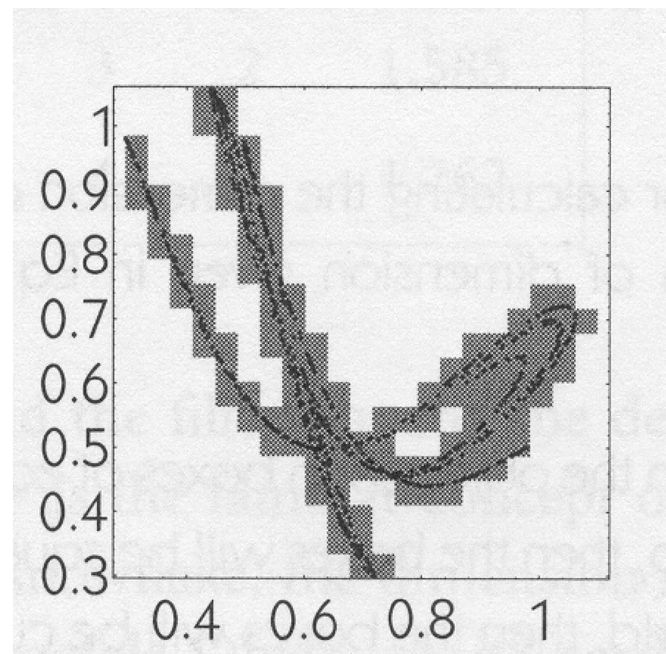
Treat the trajectory as a probability cloud.

- Quantify the information content of the probability.

Treat the trajectory as an object

- Quantify the shape, e.g., scaling properties.





The State Need Not be Numerical: Symbolic Dynamics

- Can assign a “letter” to each measurement, e.g. U, S, D depending on whether the IBI went “up”, “down” or “steady.”
- Embedding amounts to looking at the “words” of consecutive letters in the signal.
- Concepts of information and state transition still apply.

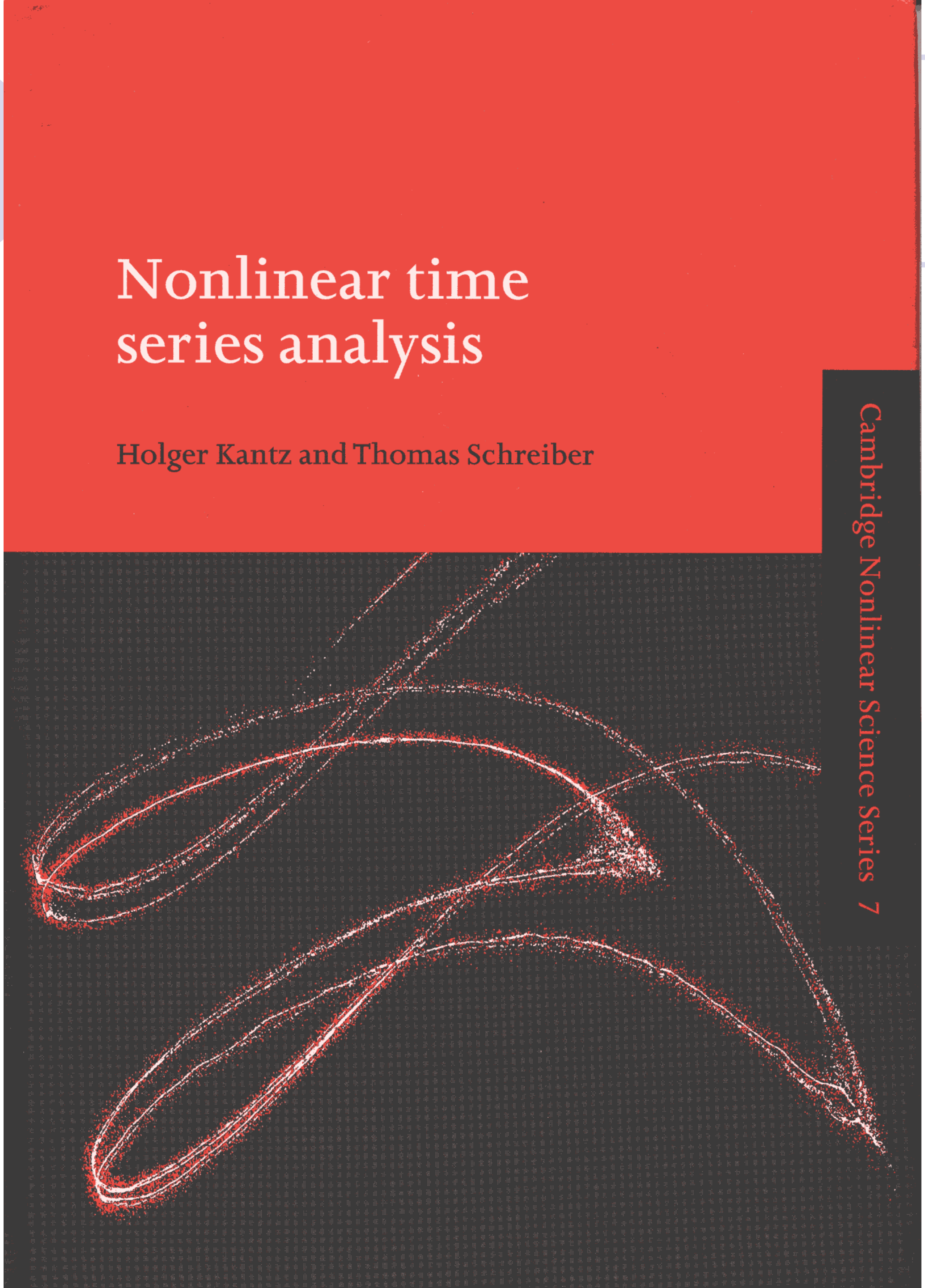
Conclusion

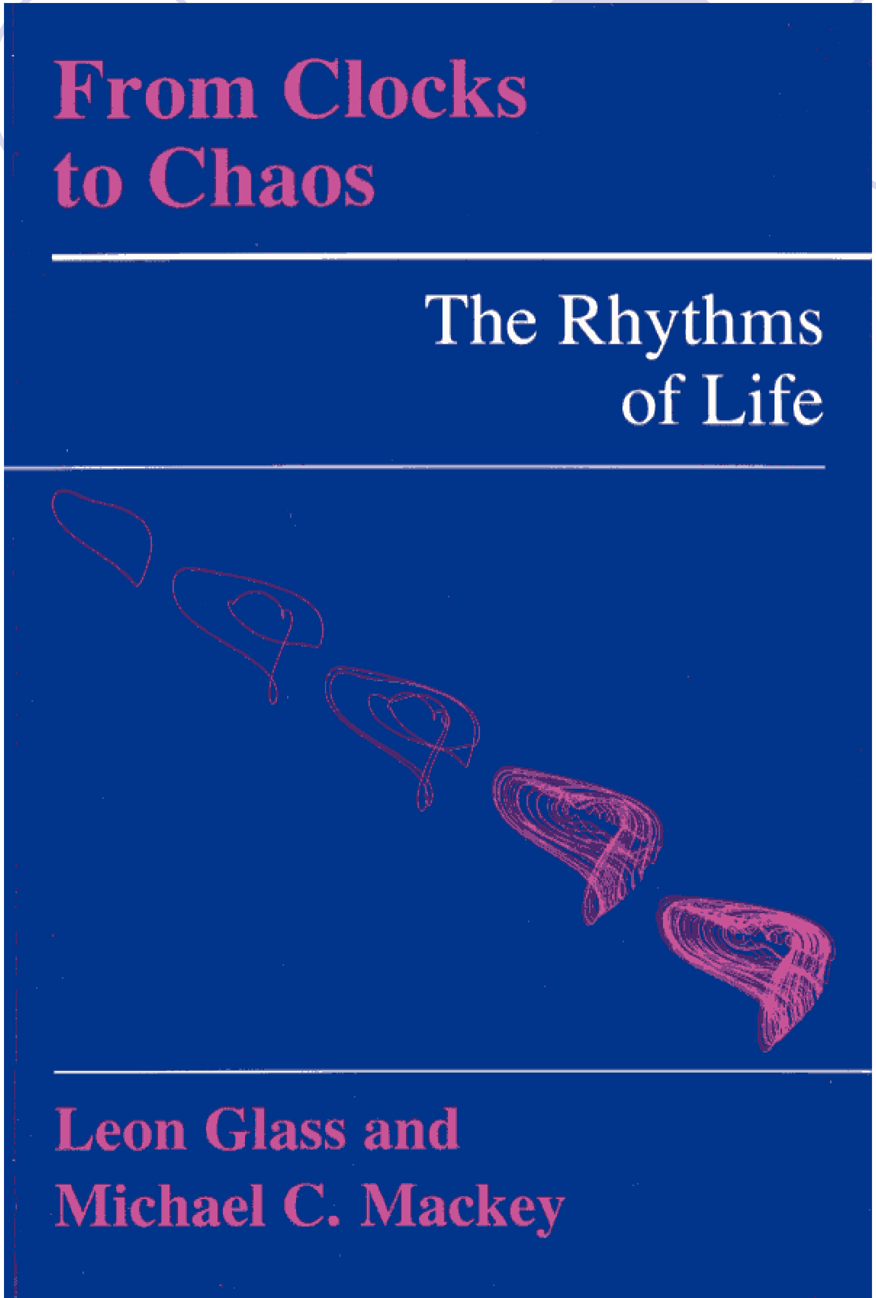
- Simple systems
  - simple behavior
  - complex-looking behavior
- Complex systems
  - simple behavior
  - complex-looking behavior
  - complex behavior

We're now pretty good at extracting information from simple signals via linear analysis and complex-looking signals via nonlinear analysis.

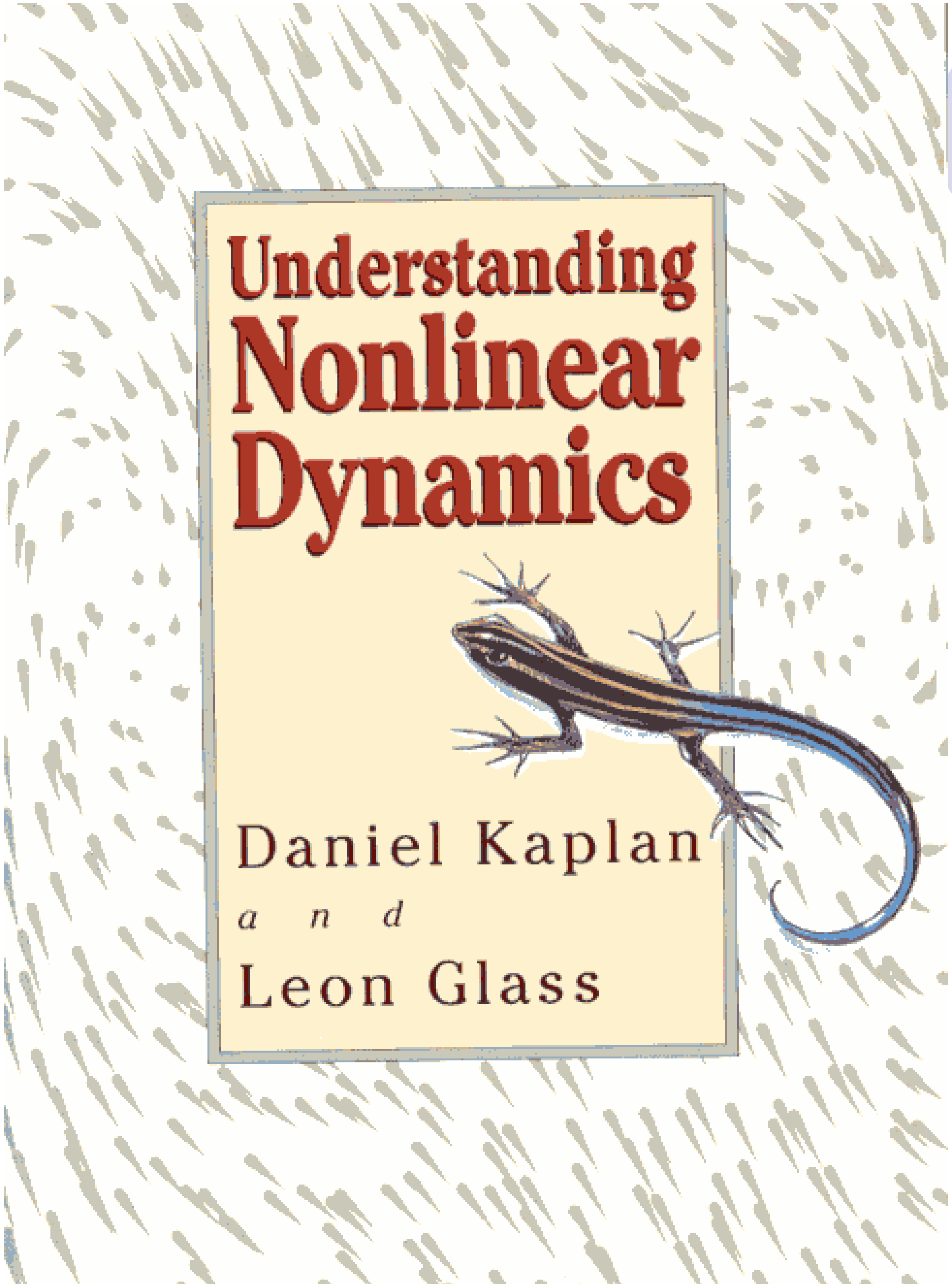
But we're not yet so good at dealing with genuinely complex signals.

Is HRV complex-looking or just complex?


Nonlinear time
series analysis
Holger Kantz and Thomas Schreiber
Cambridge Nonlinear Science Series 7



Princeton Univ. Press
1988
0-691-08496-3



Spring-Verlag NY

ISBN 0-387-94440-0